# Post-Newtonian dynamical modelling of supermassive black holes in global large-scale simulations

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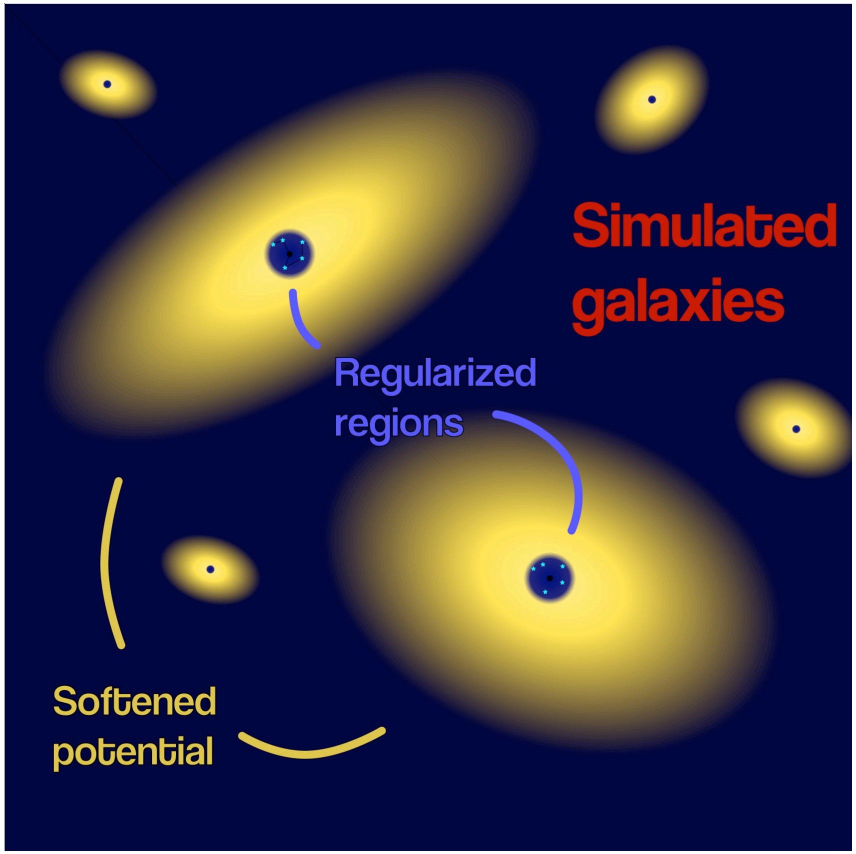
**Mannerkoski, Johansson, Pihajoki, Rantala, Naab, 2019, ApJ submitted, ArXiv: 1909.01373**
**Rantala, Johansson, Naab, Thomas, Frigo, 2019, ApJL, 872, 17**
**Rantala, Johansson, Naab, Thomas, Frigo, 2018, ApJ, 864, 113**
**Rantala, Pihajoki, Johansson, Naab, Lahén, Sawala, 2017, ApJ, 840, 53**

# 1. Motivation

- The dynamics of black holes have traditionally been studied either with 10-100 million particle softened simulations (e.g. Gadget-3) or by unsoftened direct N-body simulations restricted to ~1 million particles (e.g. Nbody-7).

- We have added a submodule to GADGET-3 that includes algorith-mically regularized regions and resolves the dynamics accurately near SMBHs with no softening.

- We can now resolve the dynamical friction, three-body interaction and gravitational wave phase self-consistently in one single simulation.

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# KETJU - Regularized Gadget: Main features

1. KETJU (chain in Finnish): An extension of Gadget-3, which includes an algorithmically regularized chain (Mikkola & Merritt 2008) module that makes two-body collisions integrable by a simple leapfrog integrator.
2. Supports multiple regularized chains, where high-resolution regularized regions can be included around every BH in the simulation.
3. Includes Post-Newtonian corrections up to order 3.5 PN (or $c^{-7}$). Includes an explicit leapfrog that account for the fact that the PN correction terms depend on the particle velocities, and possibly spins, in addition to the particle coordinates. The PN approach is valid up to ~10 Schwarzschild radii.

$$\vec{a}_{2-\mathrm{body}} = \vec{a}_{\mathrm{New}} + \sum_{k=2}^{7} c^{-k}\vec{a}_{k/2PN} + \vec{a}_S$$

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# Algorithmic chain regularization

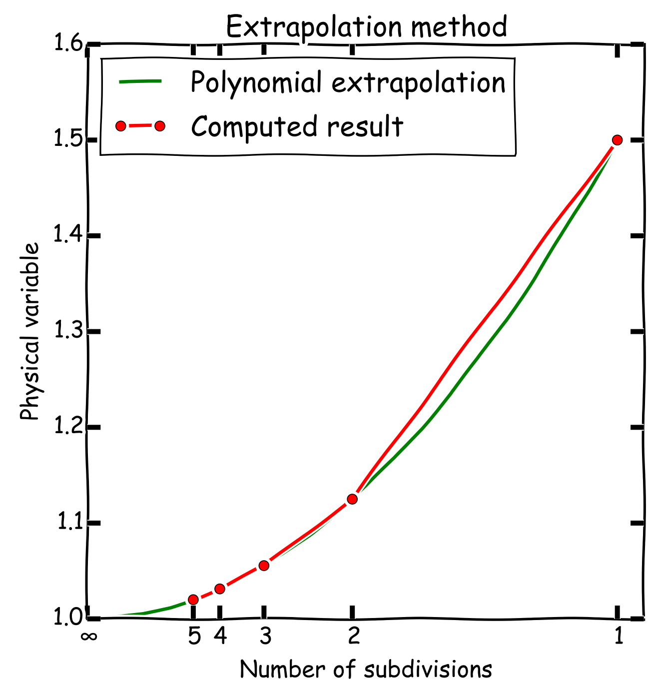
1. The dynamics in the high-resolution region is regularized through a time transformation that avoids force divergences and allows even for particle collisions (Mikkola & Tanikawa 1999, Preto & Tremaine 1999).
2. The particles are organized into a chain and in the calculation inter-particle vectors are used which significantly reduces round-off errors.
3. Particles in the chain are integrated using the Bulirsch-Stoer extrapolation method, in which a large number (~10s) substeps are taken during a full Gadget timestep resulting in good convergence.

Define $t \mapsto s$ by

$$\begin{aligned} \mathrm{d}s &= [\alpha(T+B) + \beta\omega + \gamma]\,\mathrm{d}t \\ &= (\alpha U + \beta\Omega + \gamma)\,\mathrm{d}t, \end{aligned}$$

where $\alpha, \beta, \gamma \in \mathbb{R}$, and

$$\begin{aligned} T &= \sum_i \frac{1}{2} m_i \|\vec{v}_i\|^2 \quad \text{kinetic energy,} \\ U &= \sum_i \sum_{j>i} \frac{G m_i m_j}{\|\vec{r}_{ij}\|} \quad \text{force function,} \\ B &= -T + U \quad \text{binding energy,} \\ \Omega &= \text{arbitrary function of } \vec{r}_i, \\ \dot{\omega} &= \sum_i \nabla_{\vec{r}_i}\Omega \cdot \vec{v}_i. \end{aligned}$$



# Chain construction

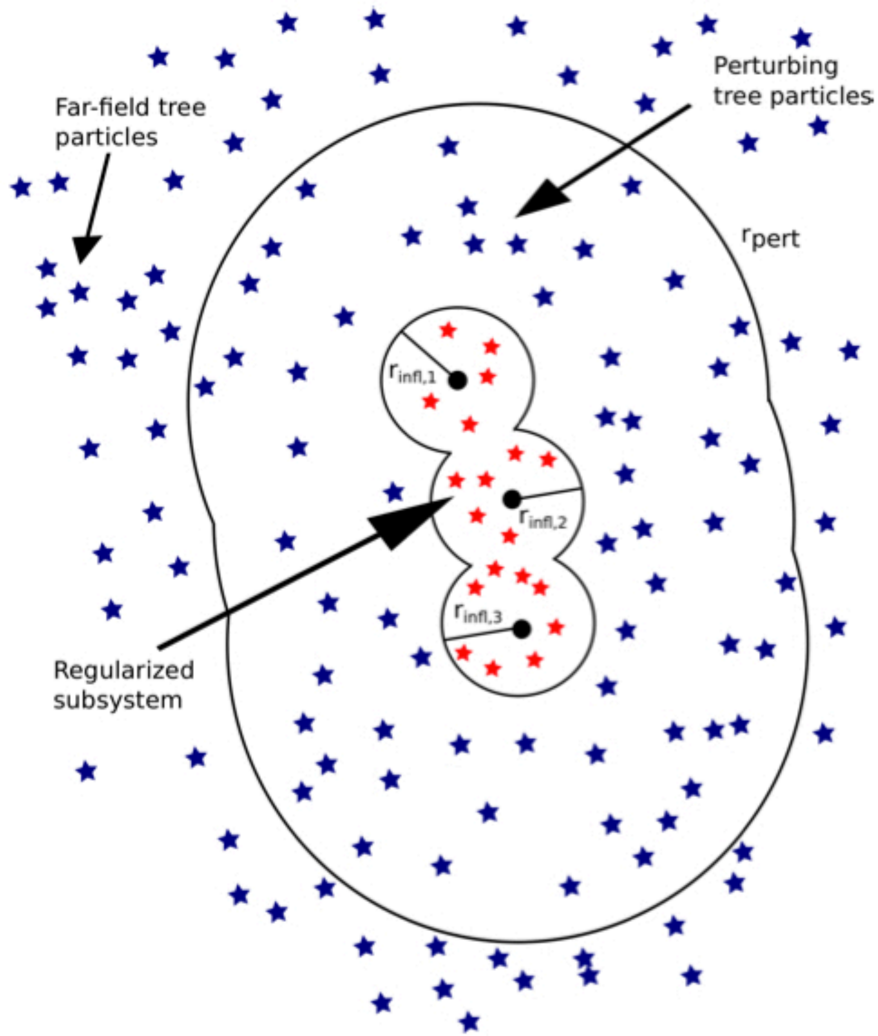
1. Chain particles: All the SMBH particles and stellar particles that lie within the influence radius of the SMBHs. Typically $r_{infl}$~10-30 pc.

$$r_{\mathrm{infl}} = \lambda \times \frac{M_{\mathrm{BH}}}{10^{10} M_{\odot}} \mathrm{kpc}$$

2. Perturber particles: Simulation particles, which induce strong tidal perturbations on a chain system. Typically $r_{pert}$=2x$r_{infl}$

$$r < r_{\mathrm{pert}} = \gamma \times r_{\mathrm{infl}} \left( \frac{m}{M_{\mathrm{BH}}} \right)^{1/3}$$

3. Tree particles: Other particles that do not reside near any of the SMBHs act as ordinary GADGET-3 particles.

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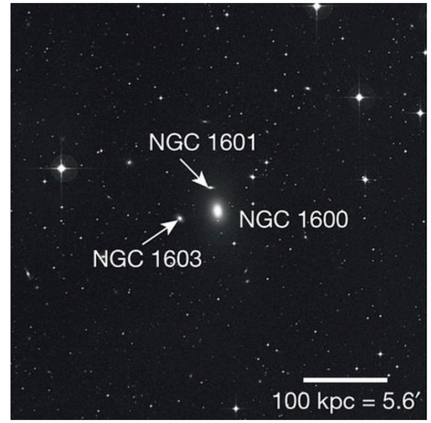
# 2. Formation of cored galaxies

Rantala, PHJ et al. 2018

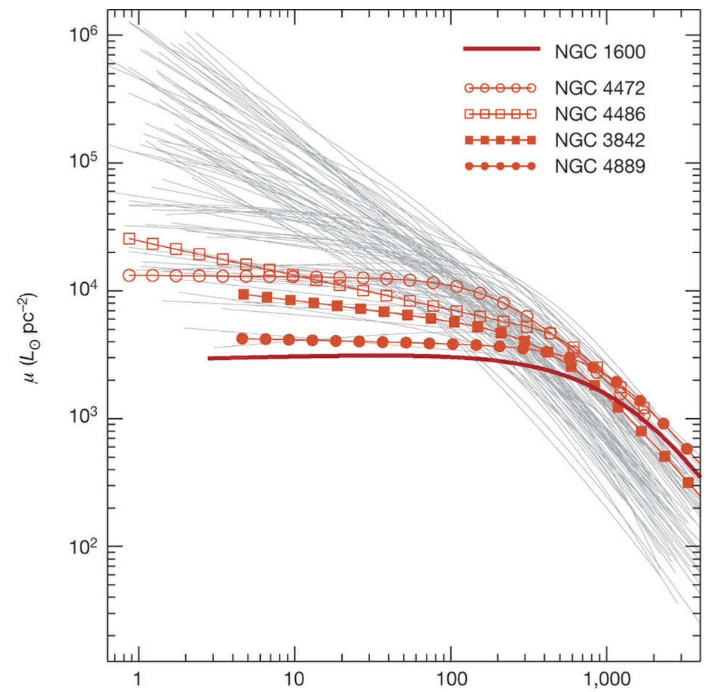
- Core ellipticals exhibit large cores with nearly constant surface brightness. Typically very massive, slowly rotating and have boxy isophotes.

- Probably formed through a dry (gas-poor) merger between two massive early-type galaxies and scouring of the core by the dynamical evolution of a SMBH binary.




- NGC 1600 is an extreme example of a cored galaxy. (Thomas et al. 2016).

$$M_{\rm BH} = (1.7 \pm 0.15) \times 10^{10} M_\odot$$

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# Initial conditions and simulations

- Our collisionless initial conditions are modelled using isotropic Dehnen profiles ($\gamma$=1.5 or $\gamma$=1.0) for the stars and $\gamma$=1.0 for the dark matter, including a central SMBH.

$$\rho(r) = \frac{(3-\gamma)M}{4\pi} \frac{a}{r^{\gamma}(r+a)^{4-\gamma}}$$

- We simulate major mergers to describe the final dry major merger that NGC 1600 likely experienced.
- High numerical resolution for N-body type of simulation.

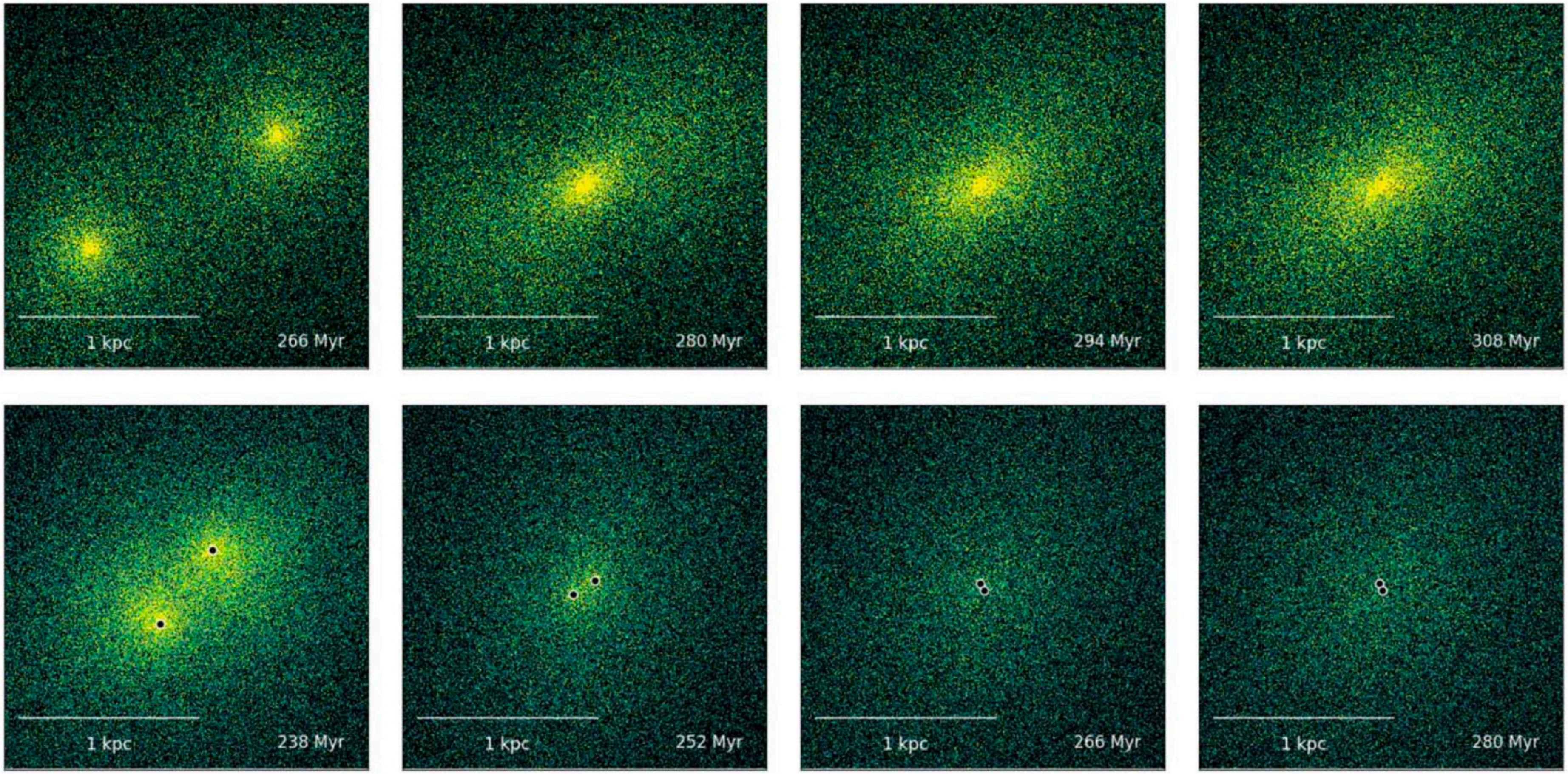
| Parameter | Symbol | Value |
|---|---|---|
| Stellar mass | $M_\star$ | $4.15 \times 10^{11}\ M_\odot$ |
| Effective radius | $R_e$ | 7 kpc |
| DM halo mass | $M_{DM}$ | $7.5 \times 10^{13}\ M_\odot$ |
| DM fraction | $f_{DM}$ | 0.25 |
| Number of stellar particles | $N_\star$ | $4.15 \times 10^6$ |
| Number of DM particles | $N_{DM}$ | $1.0 \times 10^7$ |

| Progenitor | $\gamma$ | $M_\bullet$ | Progenitor | $\gamma$ | $M_\bullet$ |
|---|---|---|---|---|---|
| $\gamma$-1.0-BH-0 | 1.0 | - | $\gamma$-1.5-BH-0 | 1.5 | - |
| $\gamma$-1.0-BH-1 | 1.0 | $8.5 \times 10^8\ M_\odot$ | $\gamma$-1.5-BH-1 | 1.5 | $8.5 \times 10^8\ M_\odot$ |
| $\gamma$-1.0-BH-2 | 1.0 | $1.7 \times 10^9\ M_\odot$ | $\gamma$-1.5-BH-2 | 1.5 | $1.7 \times 10^9\ M_\odot$ |
| $\gamma$-1.0-BH-3 | 1.0 | $3.4 \times 10^9\ M_\odot$ | $\gamma$-1.5-BH-3 | 1.5 | $3.4 \times 10^9\ M_\odot$ |
| $\gamma$-1.0-BH-4 | 1.0 | $5.1 \times 10^9\ M_\odot$ | $\gamma$-1.5-BH-4 | 1.5 | $5.1 \times 10^9\ M_\odot$ |
| $\gamma$-1.0-BH-5 | 1.0 | $6.8 \times 10^9\ M_\odot$ | $\gamma$-1.5-BH-5 | 1.5 | $6.8 \times 10^9\ M_\odot$ |
| $\gamma$-1.0-BH-6 | 1.0 | $8.5 \times 10^9\ M_\odot$ | $\gamma$-1.5-BH-6 | 1.5 | $8.5 \times 10^9\ M_\odot$ |

BH-0: no SMBHs and BH-6: Observed SMBHs in NGC 1600.

# Stellar surface densities

Rantala, PHJ et al. 2018




- Top: Merger without SMBHs. Bottom: Merger with massive SMBHs.
- The effect of core scouring by the SMBHs can clearly be seen in the surface density plot.

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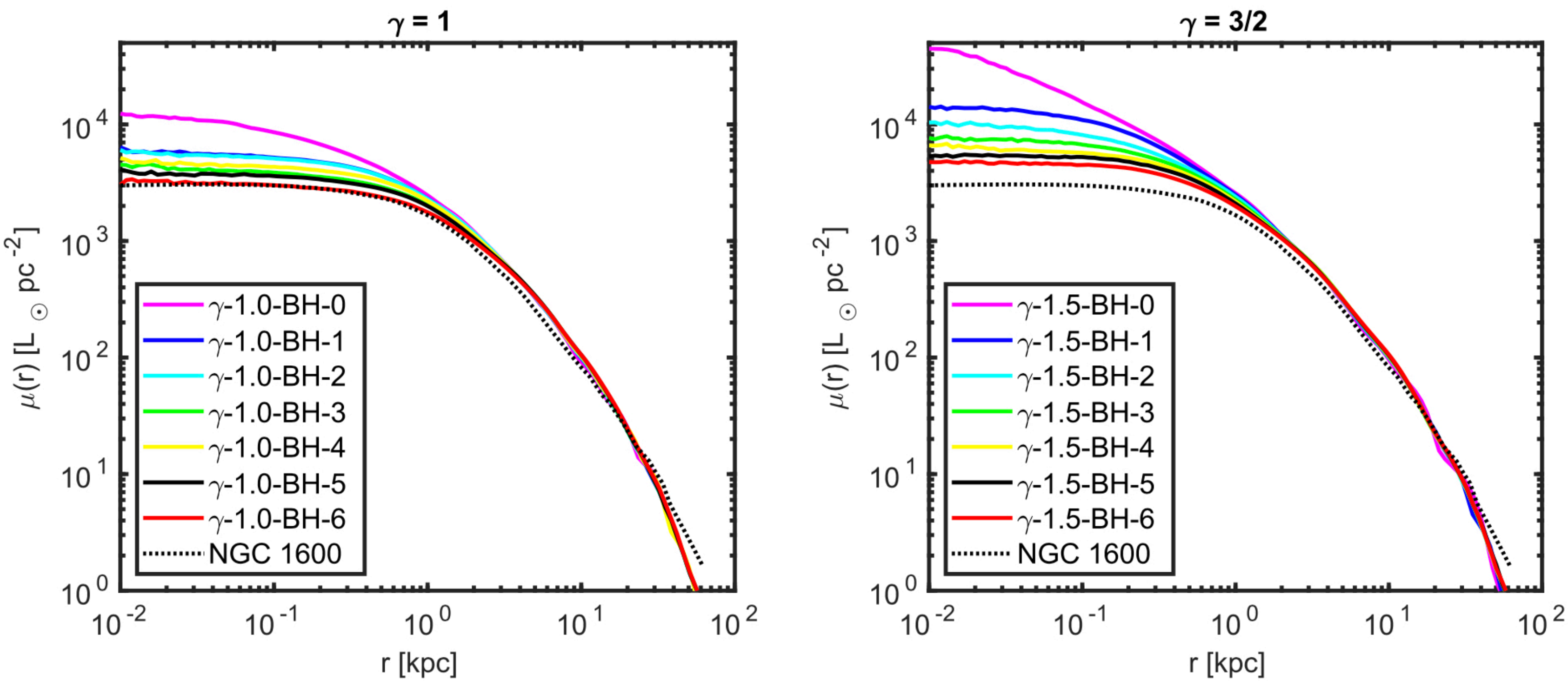
# Surface brightness profiles




- Similarly to the Thomas et al. (2016) observations we assume a constant mass-to-light ratio of $M_*/L=4.0$.
- As expected we find a systematic decrease in the surface brightness as a function of increasing BH mass (e.g. Merritt 2006).

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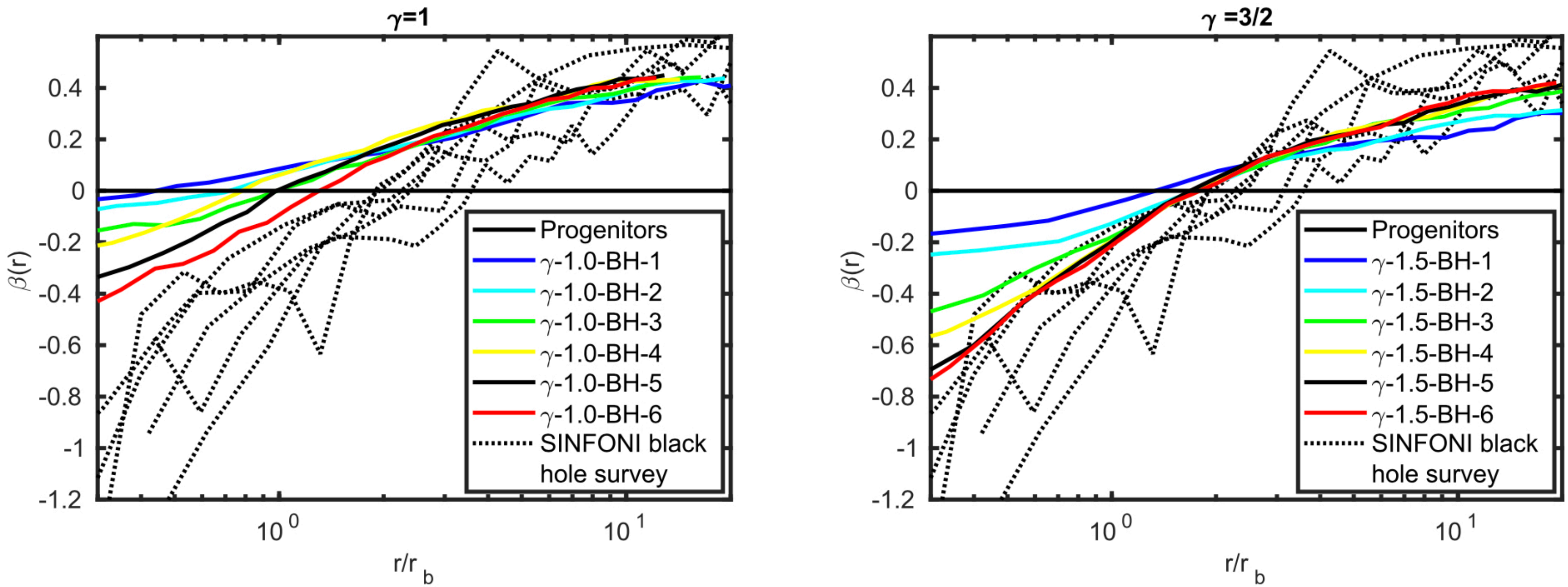
# Velocity anisotropy profiles




- We find a monotonic decrease in the central β-parameter, meaning an increasingly more tangentially biased stellar population in the core region.
- More massive BHs have larger spheres of influence -> more negative β.

$$\beta = 1 - \frac{\sigma_\theta^2 + \sigma_\phi^2}{2\sigma_{\rm r}^2} = 1 - \frac{\sigma_{\rm t}^2}{\sigma_{\rm r}^2}.$$

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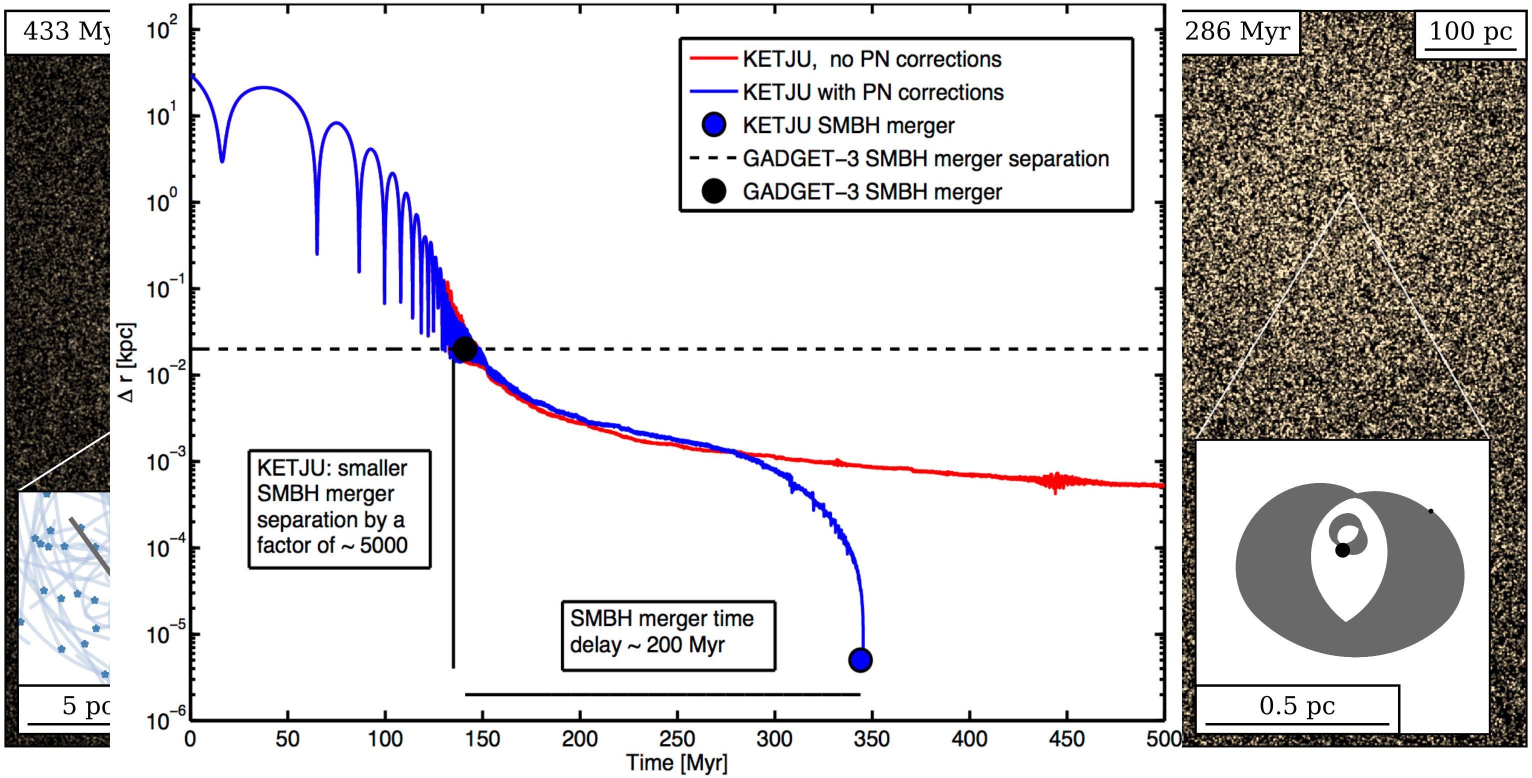
# 3. Direct Gravitational wave calculations

**Mannerkoski, PHJ et al. 2019**




- With KETJU we can resolve the dynamics of SMBHs down to separations of ~10 Schwarzschild radii.
- Simulations with large softening lengths must resort to semi-analytic models to describe the dynamics below the softening length (~700 pc for both ILLUSTRIS (Kelley et al. 2017) and the EAGLE simulation (Salcido et al. 2016)).

# Comparison to semi-analytic calculations

4 merger generations: 5:1, 6:1, 7:1, 8:1. 1:1 merger with lower SMBH masses.

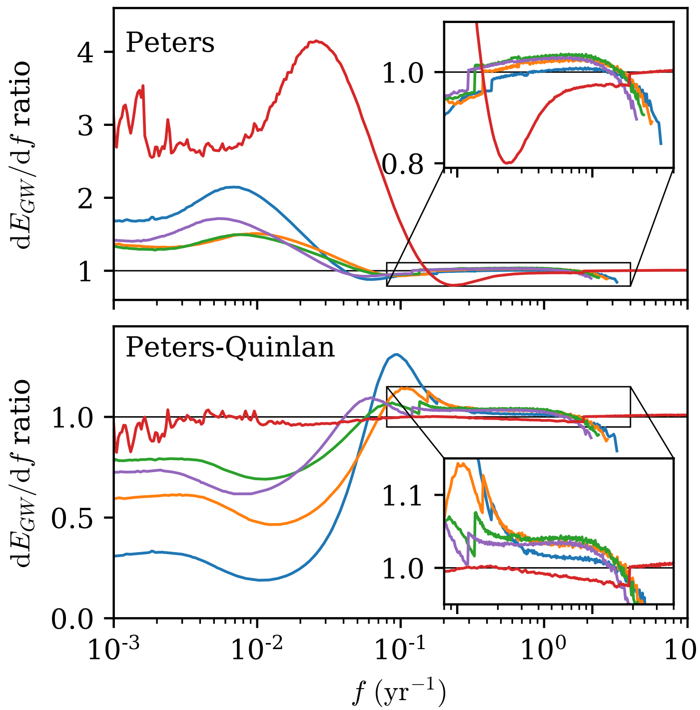
Peters & Mathews (1963) PN2.5 radiation reaction only term:

$$\left|\frac{da}{dt}\right| = \frac{64}{5}\frac{G^3 M_1 M_2 (M_1+M_2)}{c^5 a^3}\frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}}$$

Semi-Analytic Scattering model (Quinlan 1996; Sesana et al. 2006):

$$\frac{d}{dt}\left(\frac{1}{a}\right) = \frac{G\rho}{\sigma}H$$

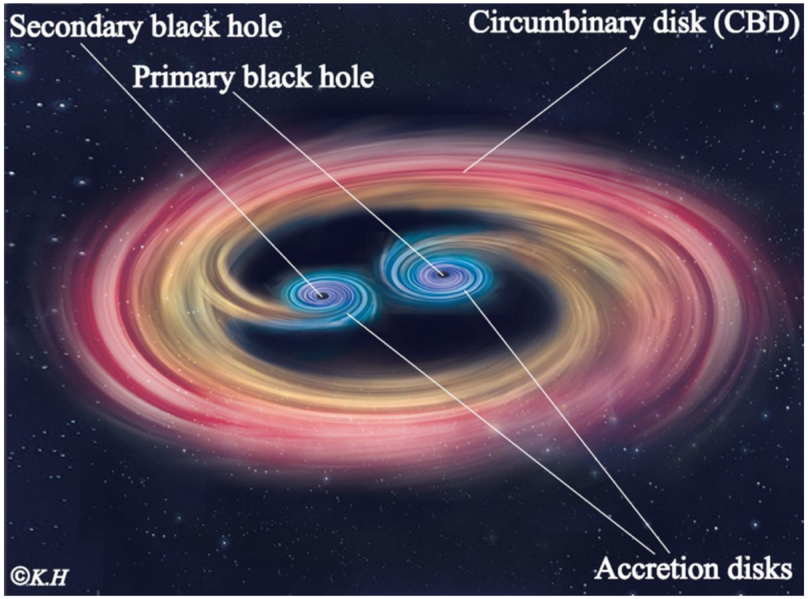
$$\frac{de}{dt} = \frac{G\rho}{\sigma}aHK$$

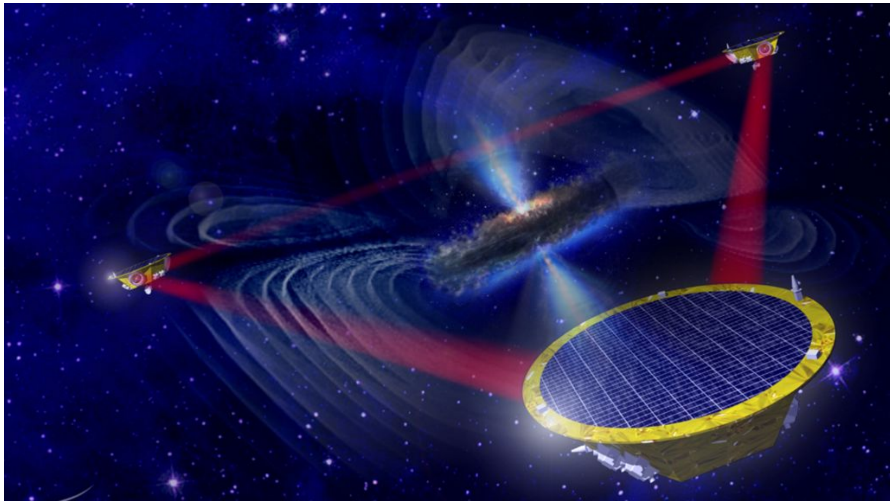



- The emitted total gravitational energy as a function of frequency can be calculated from the KETJU orbital elements and during the final KETJU timestep directly from the SMBH positions and velocities using a fast Fourier transform algorithm.
- Differences to commonly used semi-analytic models can be in excess of 10% in the Pulsar timing array bands (see insets)

# 4. KETJU and gas physics

- Since KETJU interfaces with GADGET, SPH can be used to resolve the large-scale hydrodynamics of the gas.
- The circumbinary disc is directly resolvable, but the individual accretion discs must be treated with a subresolution model.
- The prolonged binary phase will require improved accretion models compared to the standard Bondi-Hoyle prescription.
- Accurate dynamics combined with detailed hydrodynamics will be important for making accurate model predictions for LISA.



# Summary

- The KETJU code is a version of Gadget includes an algorithmically regularized chain module that makes two-body collisions integrable by a simple leapfrog integrator.
- Cores form rapidly on the order of the crossing timescale by SMBH binary evolution and the velocity distribution becomes increasingly more tangential over a longer timescale.
- Accurate calculations of the GW signals show that the differences to semi-analytic models can be >10%.
- LISA will be most sensitive to GW signals from SMBHs with masses in range $10^6$-$10^7$ $M_{\odot}$, thus modelling the accurate small-scale dynamics simultaneously with gas physics will be important.

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