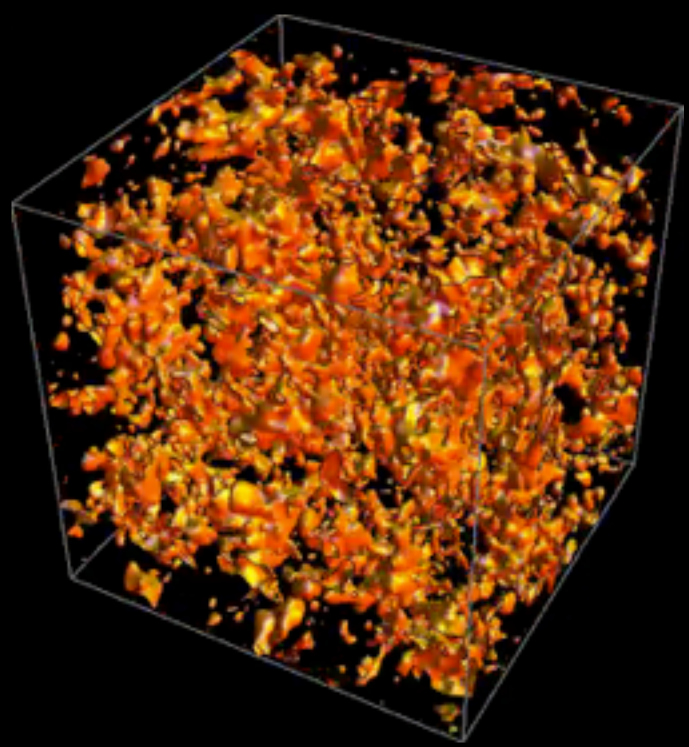
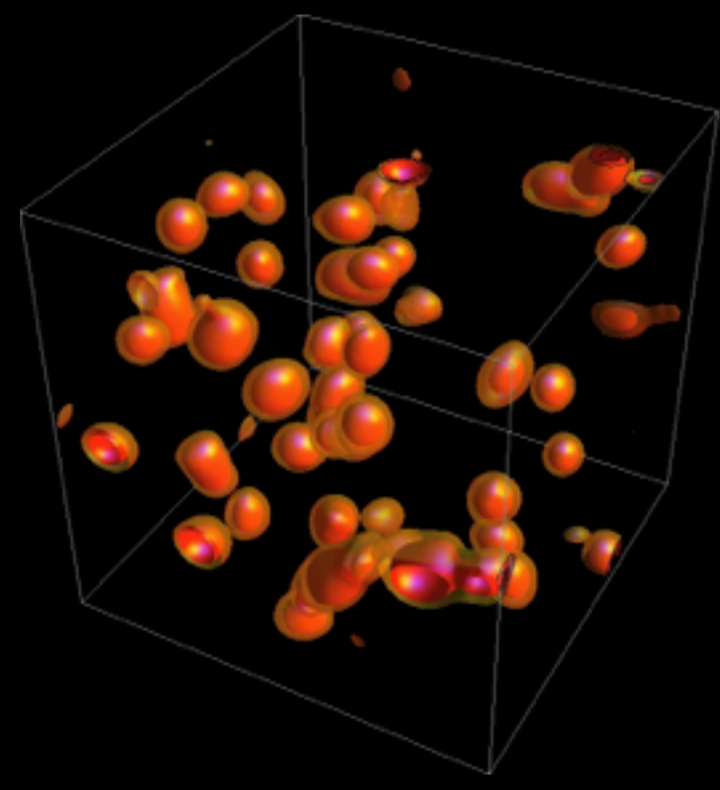
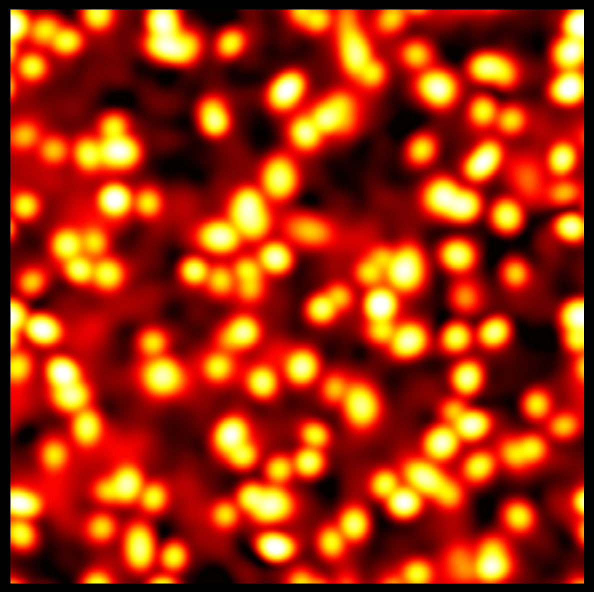
Nonlinear Dynamics of Cosmological Scalar Fields







Mustafa A. Amin

+ **Kaloian Lozanov**, P. Mocz, T. Smith
+ earlier collaborators (Easther, Poulin +)

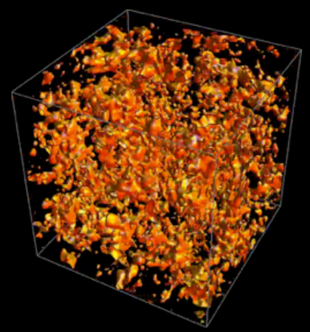
Nonlinear Dynamics of Cosmological Scalar Fields

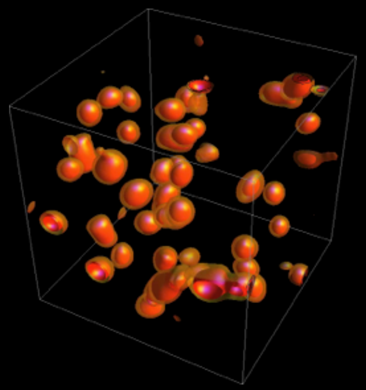
theoretical/numerical results

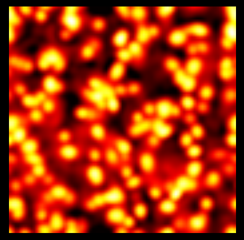
1. instability in oscillating fields
2. formation of solitons
3. eq. of state

obs. implications

1. gravitational waves
2. structure formation
3. expansion history







why study nonlinear dynamics cosmological (scalar) fields ?

why scalar fields

- inflation & end of inflation (reheating)
- Higgs dynamics
- dark matter (axions / fuzzy dm)
- late & early dark energy

why nonlinear?

- linearity is *not* the norm
- provides many, scale-dependent predictions for observations

* sometimes necessary, sometimes easiest thing to start with when we don't know better

result 1: instability in oscillating fields

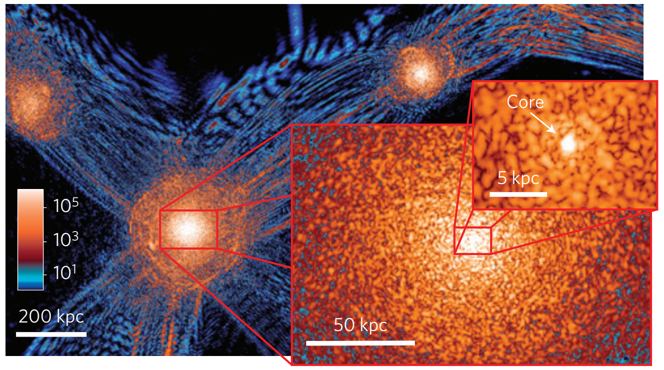
oscillating, cosmologically relevant, (almost) homogeneous scalar fields are unstable to spatial perturbations

Khlopov, Malomed, Zeldovich (1985)
Johnson & Kamionkowski (2009)

* there are timescales associated with the instability, typically the longest is Hubble time scale

why are dynamics of oscillating cosmological fields nonlinear?

- **gravity**
  - fields cluster, Hubble time scales



Schive et. al (2014)

- **self-interactions**
  - fields can cluster/become inhomogeneous
  - (can be) much faster than Hubble (due to *self*-resonance)

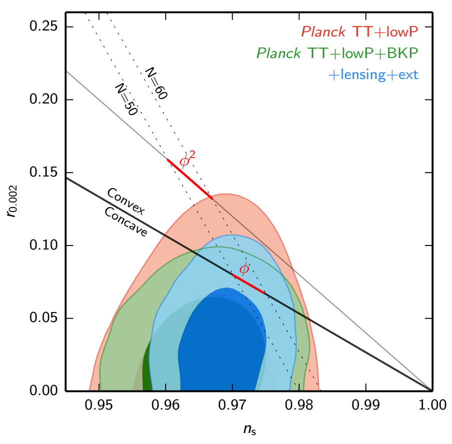
Review 1410.3808

Kofman, Linde, Starobinsky (1994)

*for this short talk, I will ignore interactions with other fields, which can also be important

why are dynamics of oscillating cosmological fields nonlinear?

Planck



- **gravity**
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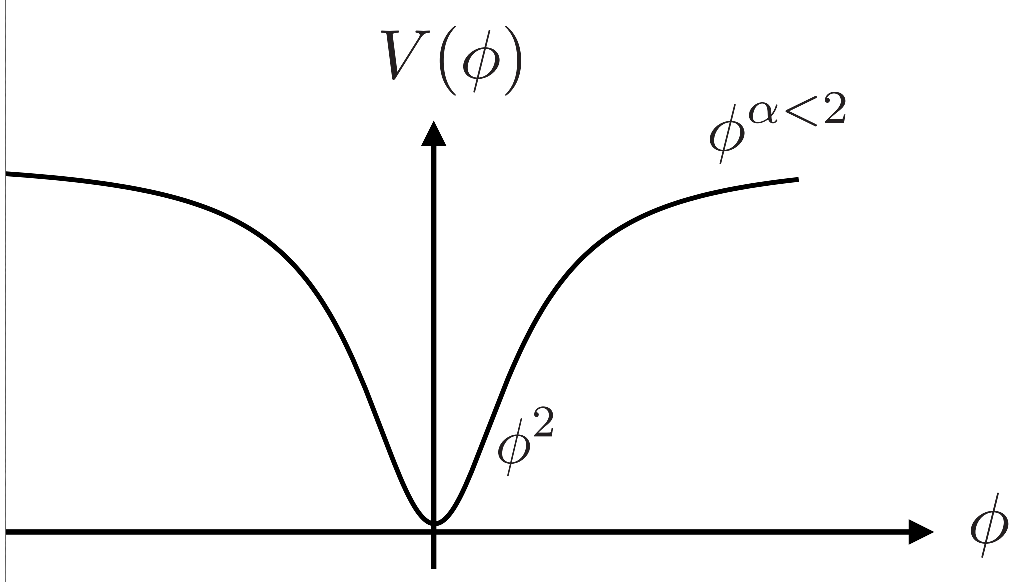
$$S = \int d^4x\sqrt{-g}\left[\frac{m_{\rm pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

$$\cancel{+U(\phi,\psi_i)}$$



Peccei & Quinn (1977), Hu, Barkana & Gruzinov (2000), Arvanitaki et. al (2009)

Starobinsky Inflation (1979), Silverstein & Westhpal (2008), Kallosh & Linde (2013)

why are dynamics of cosmological fields nonlinear?

- **gravity**
  - fields cluster, Hubble time scales

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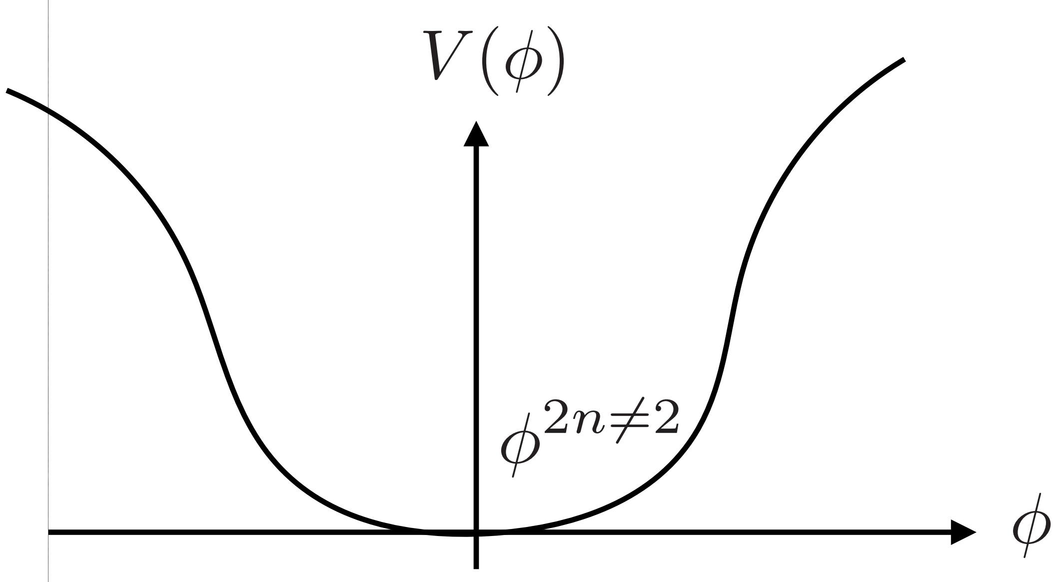
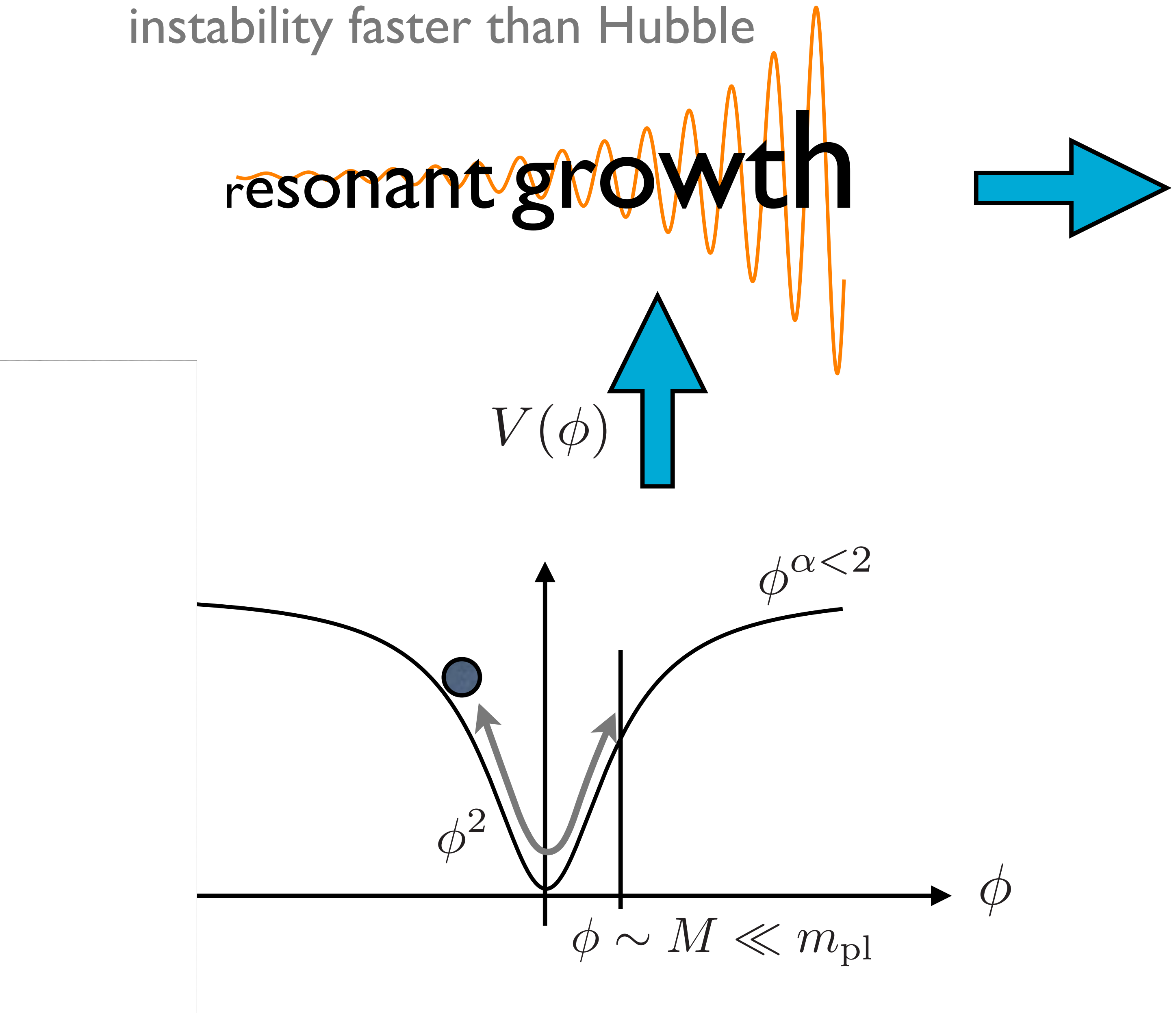
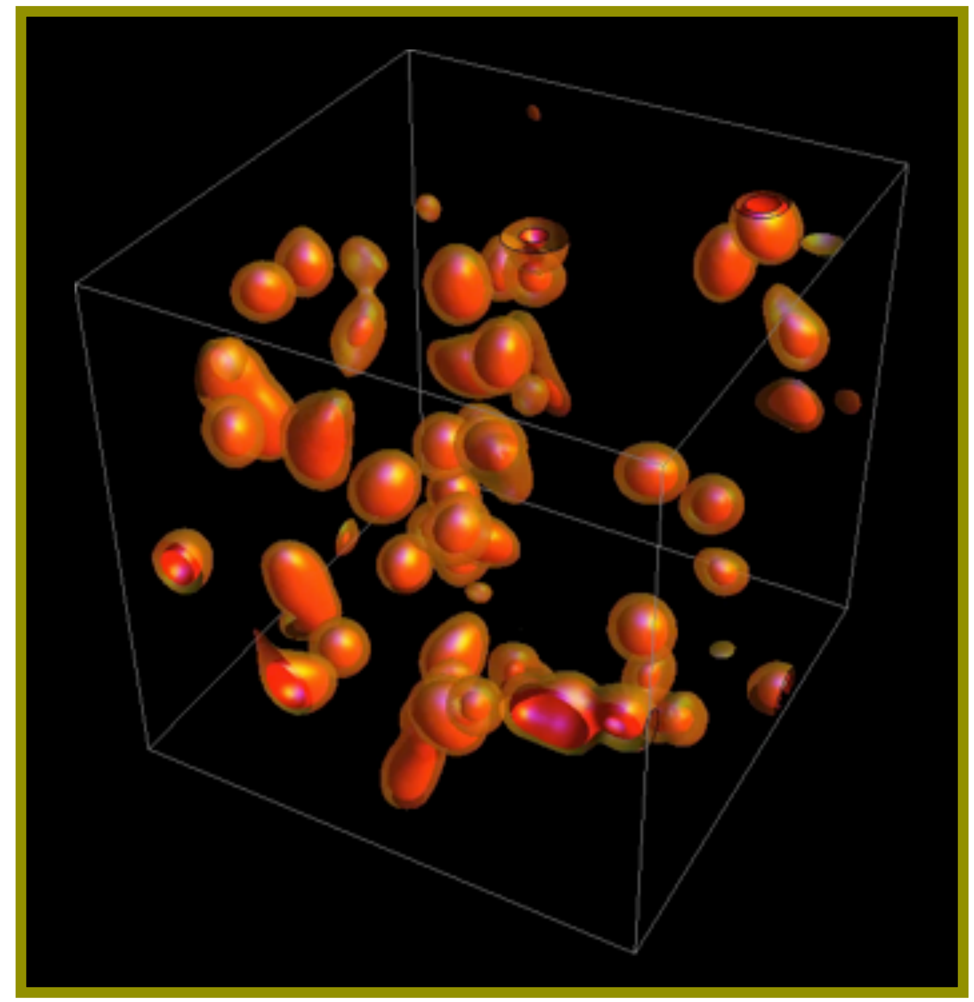


* potentials relevant for inflation (alpha-attractors, Andrei's talk), and for early DE (Mark's talk earlier)

result 2: oscillon formation (solitons)

instability faster than Hubble





MA, Easther, Finkel, Flaugher & Hertzberg (2011)
1106.3335

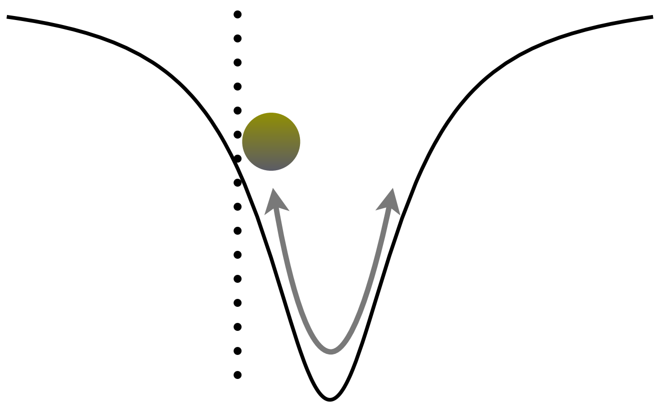
Earlier papers on oscillons eg.: Bogolubovsky & Makhanakov 1970s, Gleiser, Copeland Mueller (1990s)

dynamics of oscillating fields

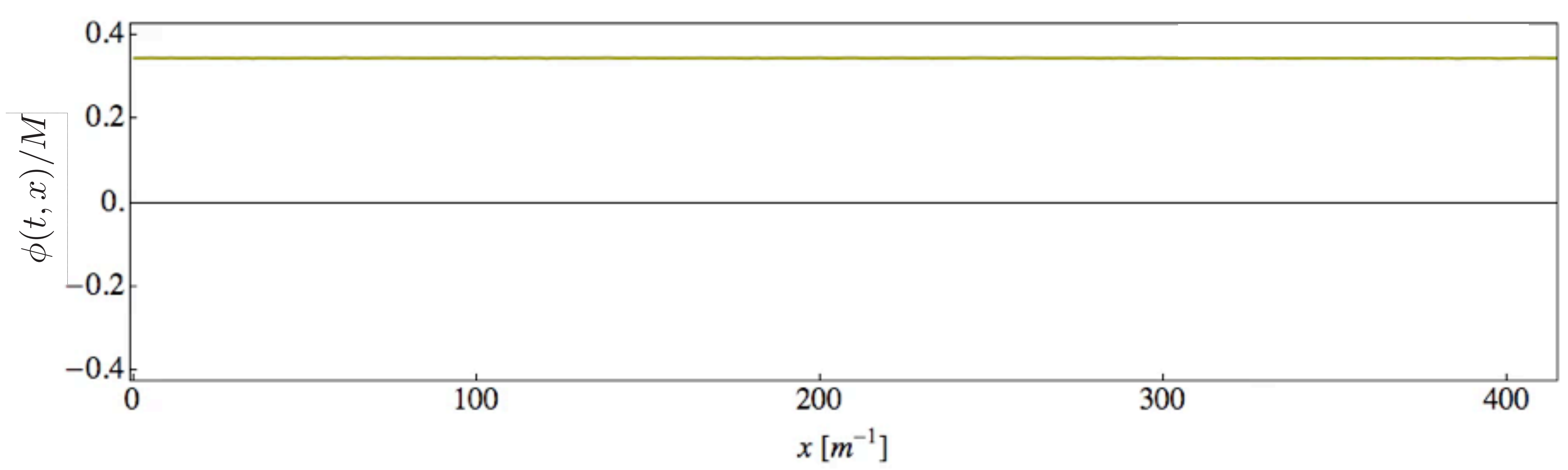
expansion ✓

self-interactions ✓

gravitational int. ✗



$$\Box\phi = V'(\phi)$$



MA (2010)
1006.3075

dynamics of oscillating fields

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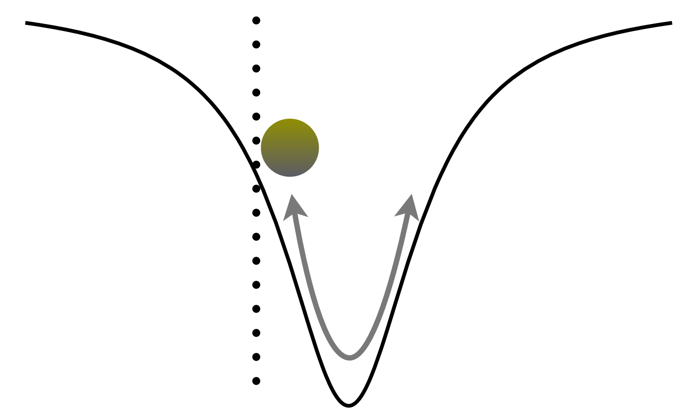

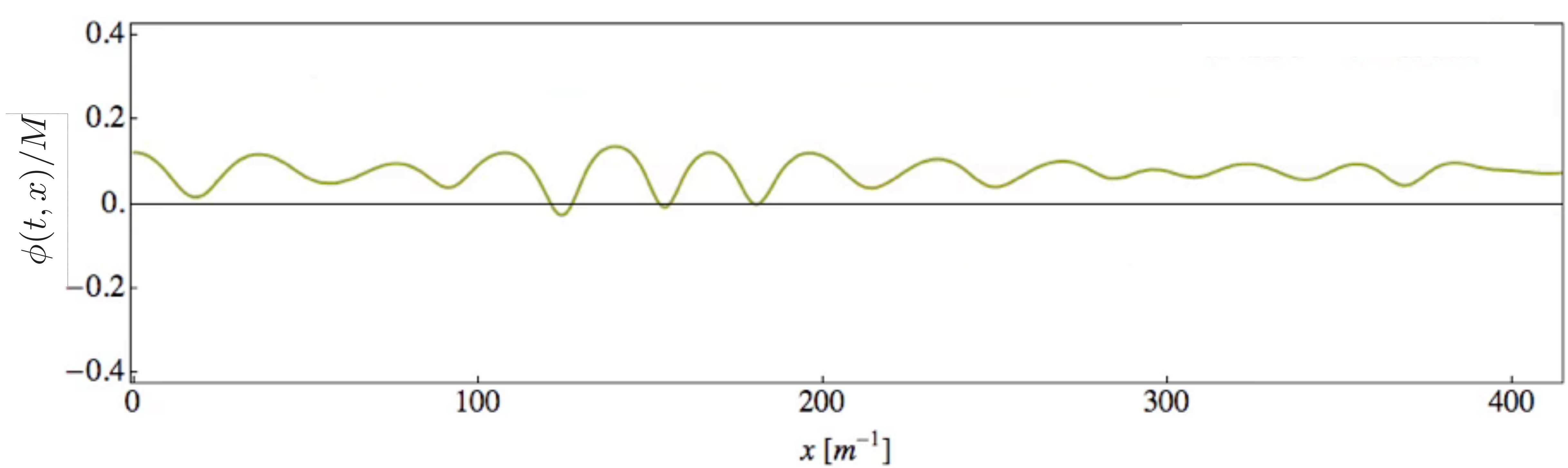

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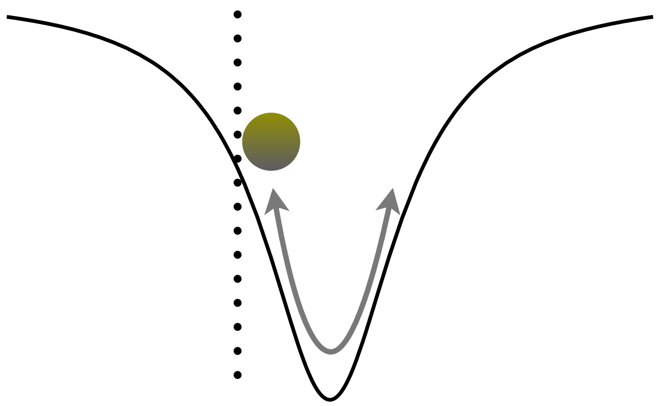

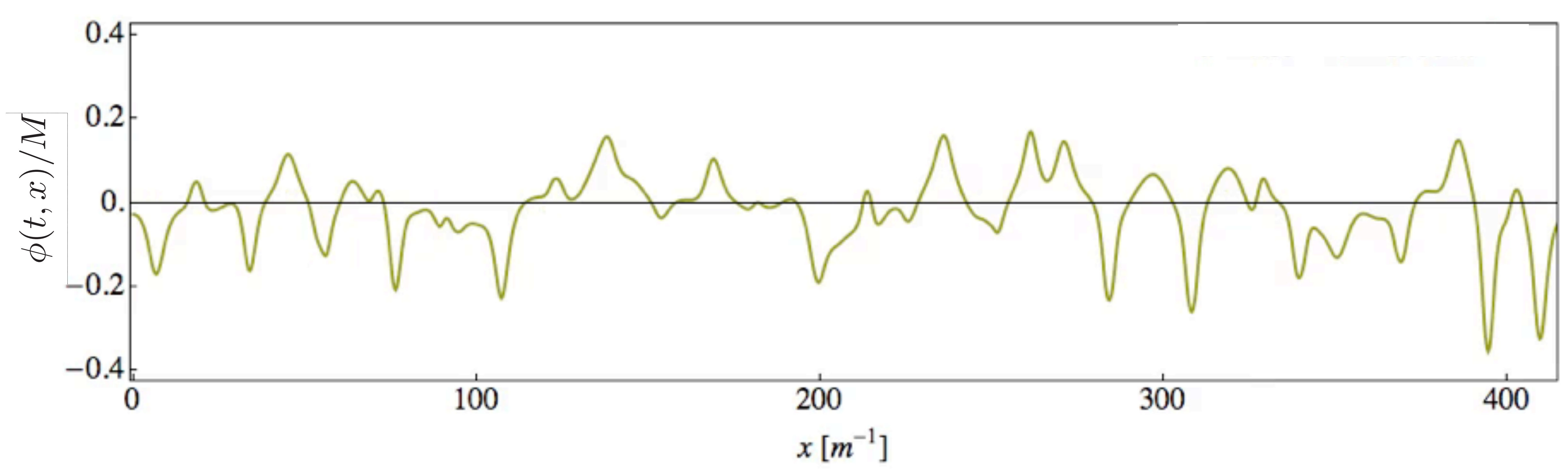

MA (2010)
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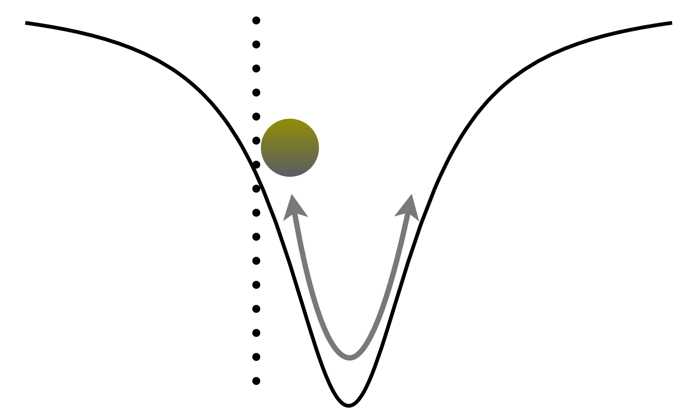
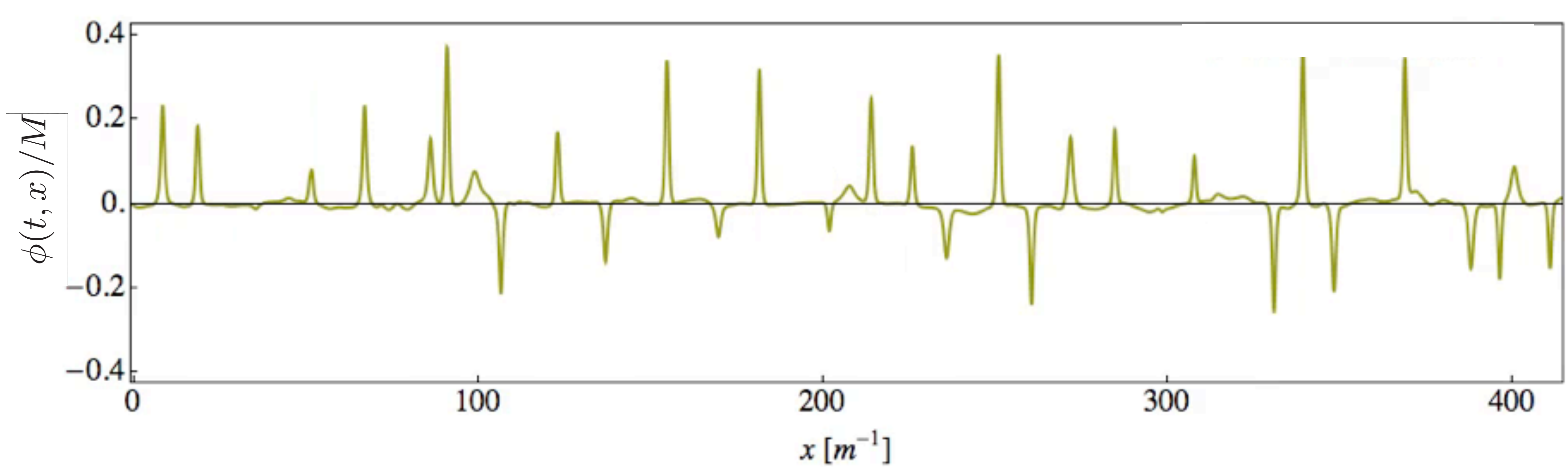




MA (2010)
1006.3075

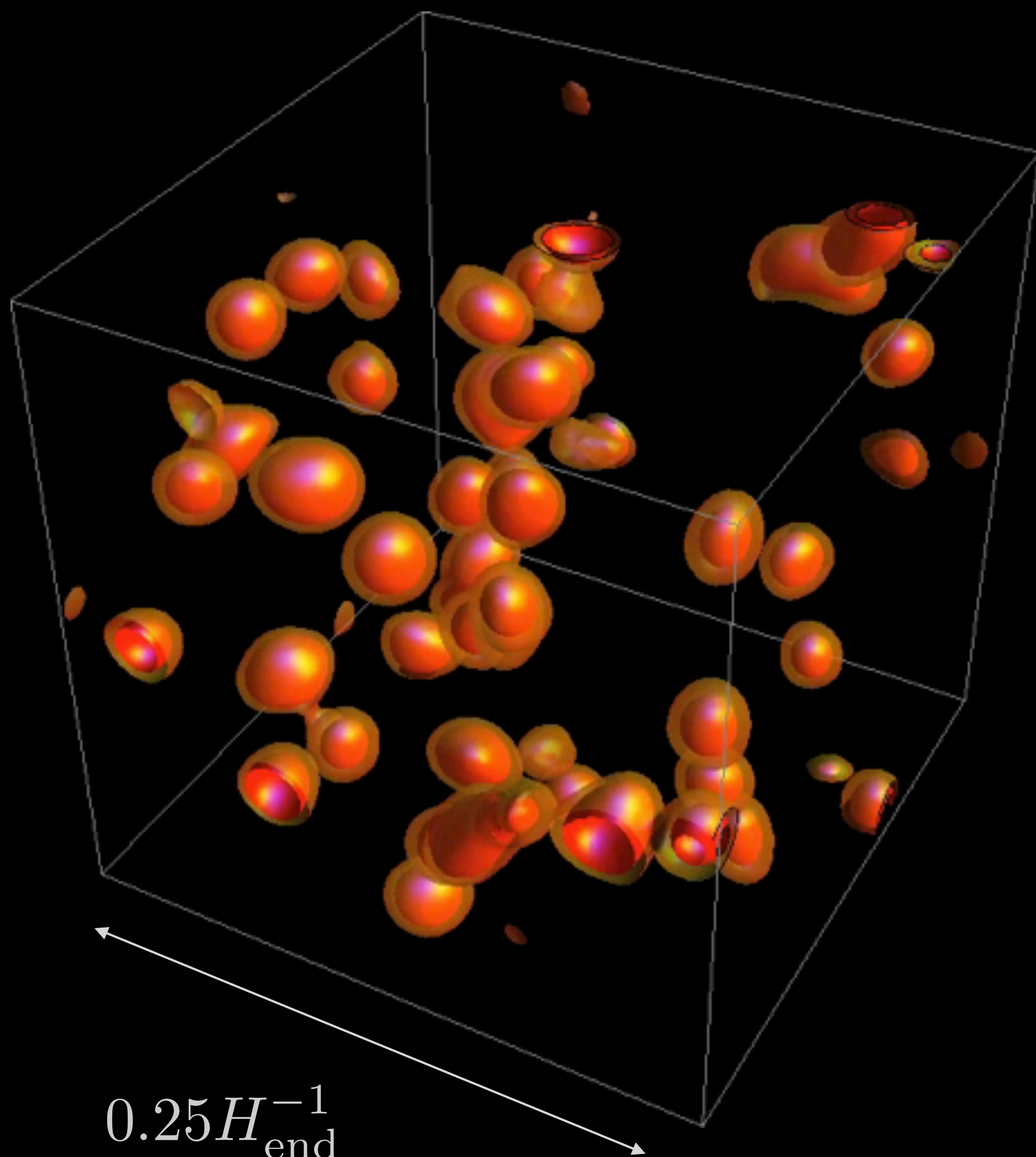
soliton formation in oscillating fields

expansion ✓

self-interactions ✓

gravitational int. ✗

solitons dominate the energy density of the field!

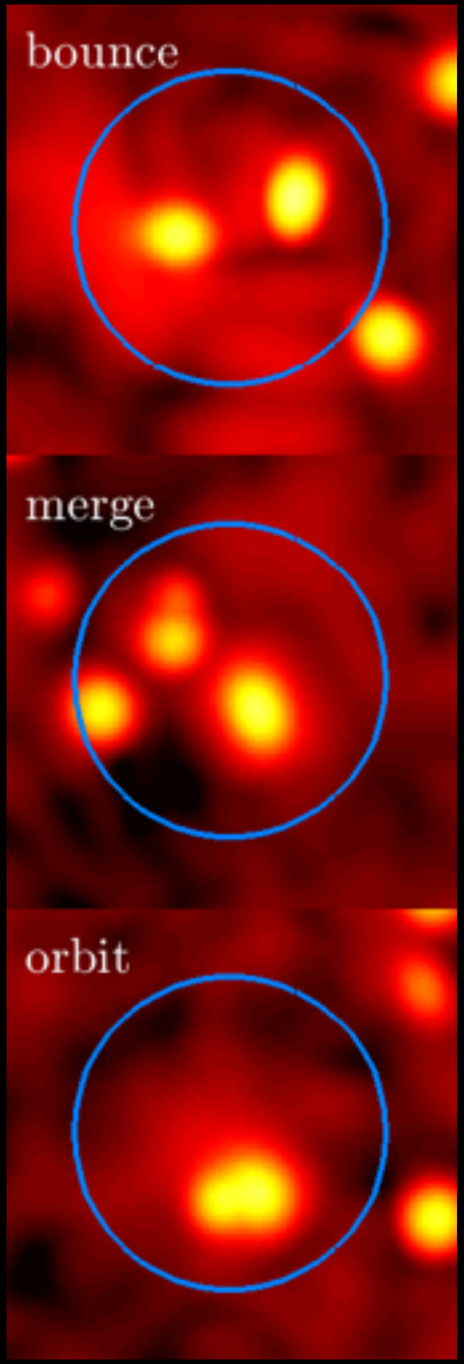


MA, Easther, Finkel, Flaugher & Hertzberg (2011)
1106.3335

self-interactions

+ gravity*

(Schrodinger-Poisson)

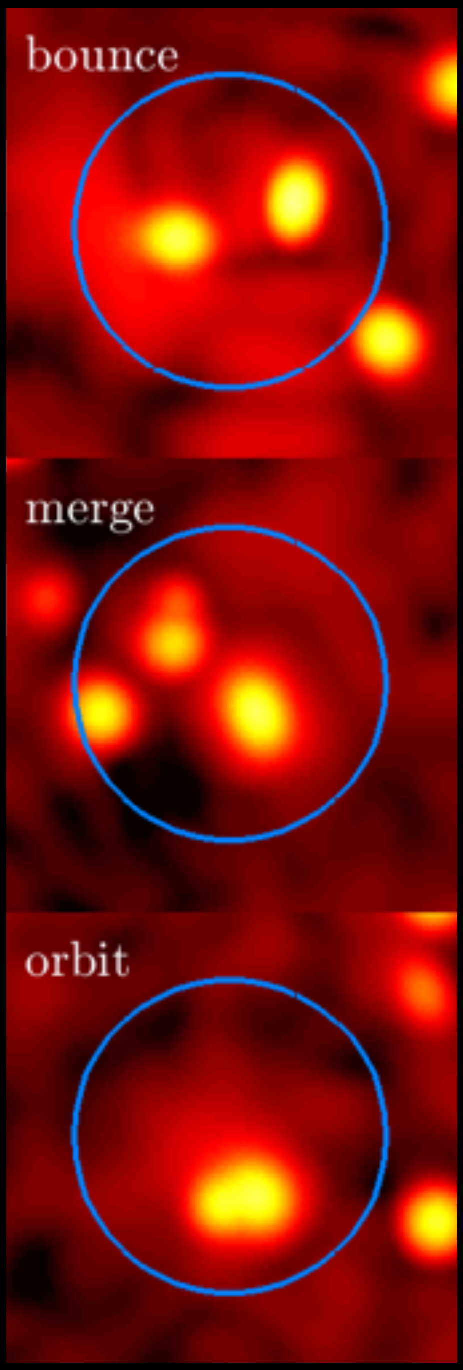


MA & Mocz (2019)
1902.07261

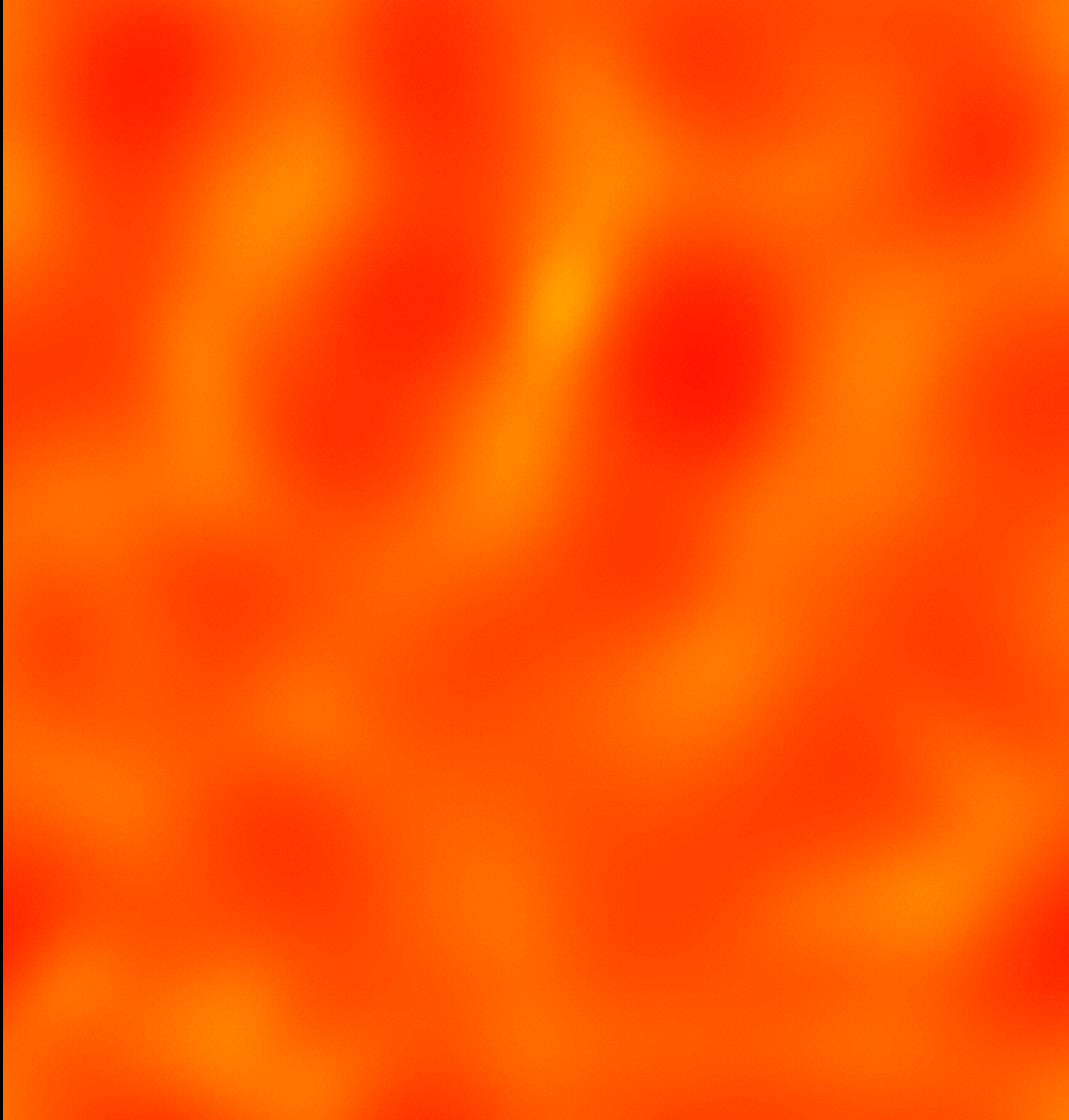
self-interactions

$+$ gravity*

(Schrodinger-Poisson)



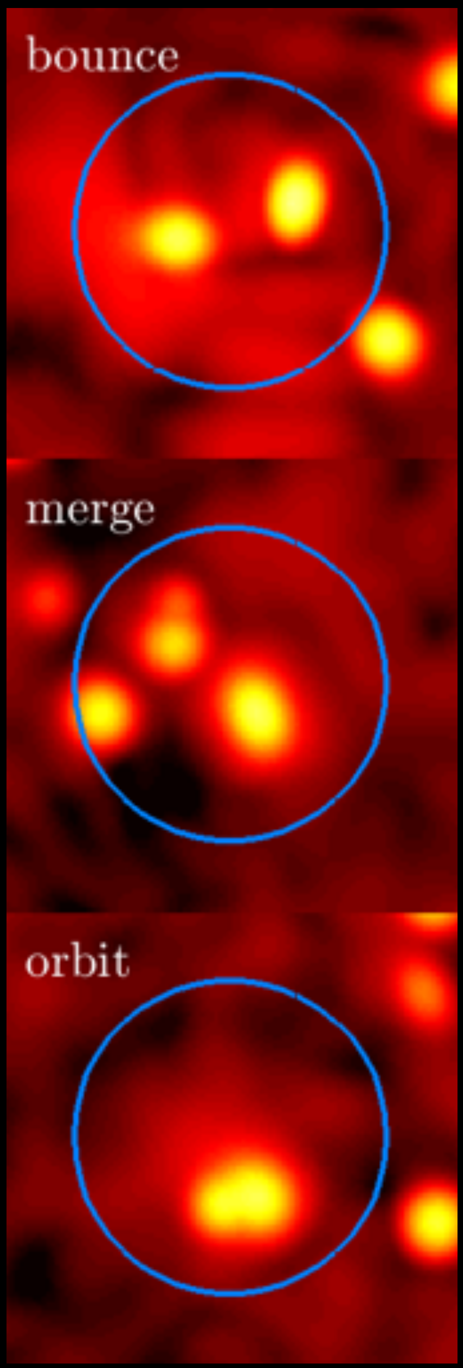
MA & Mocz (2019)
1902.07261



self-interactions

+ gravity*

(Schrodinger-Poisson)



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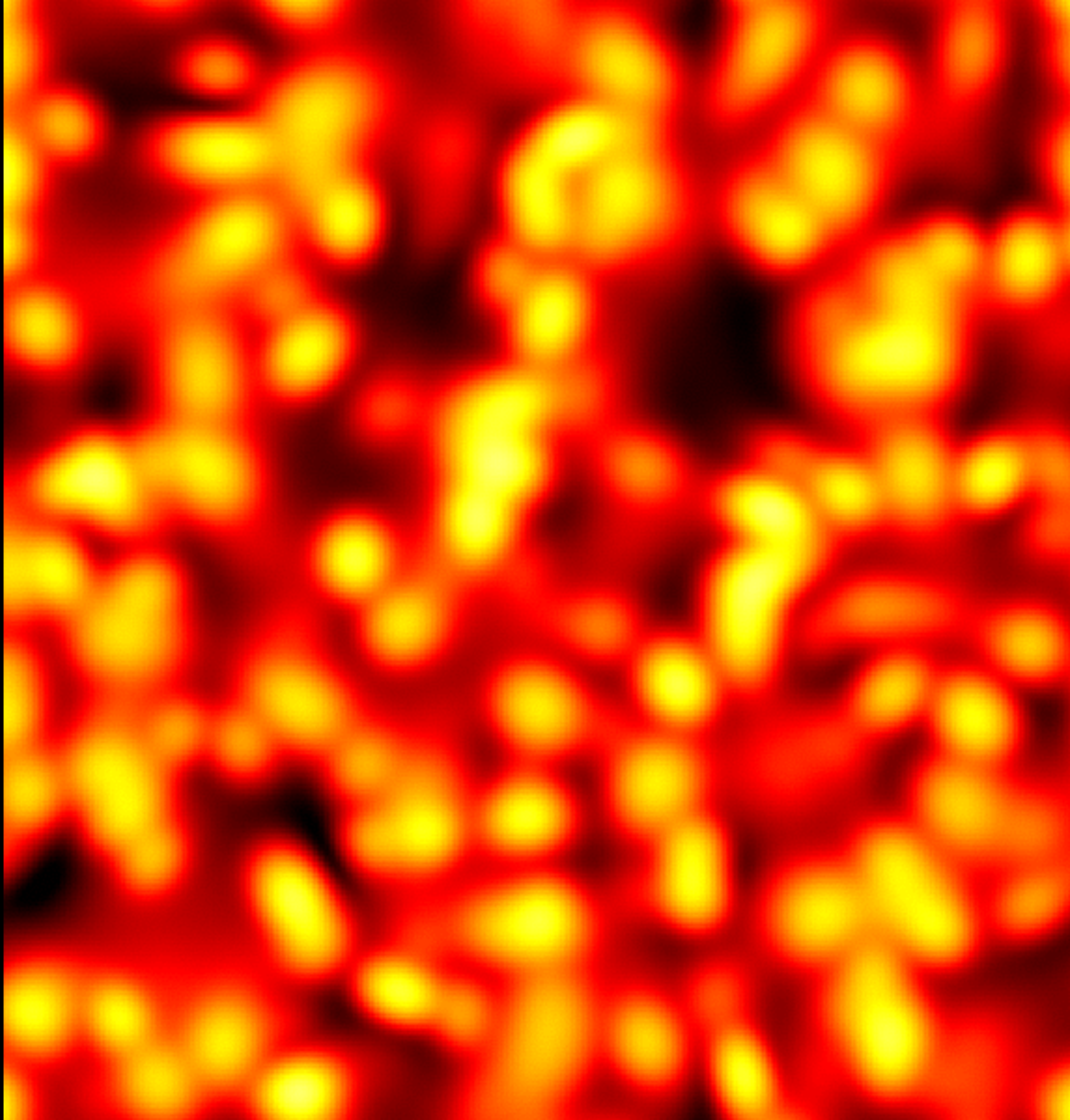

self-interactions

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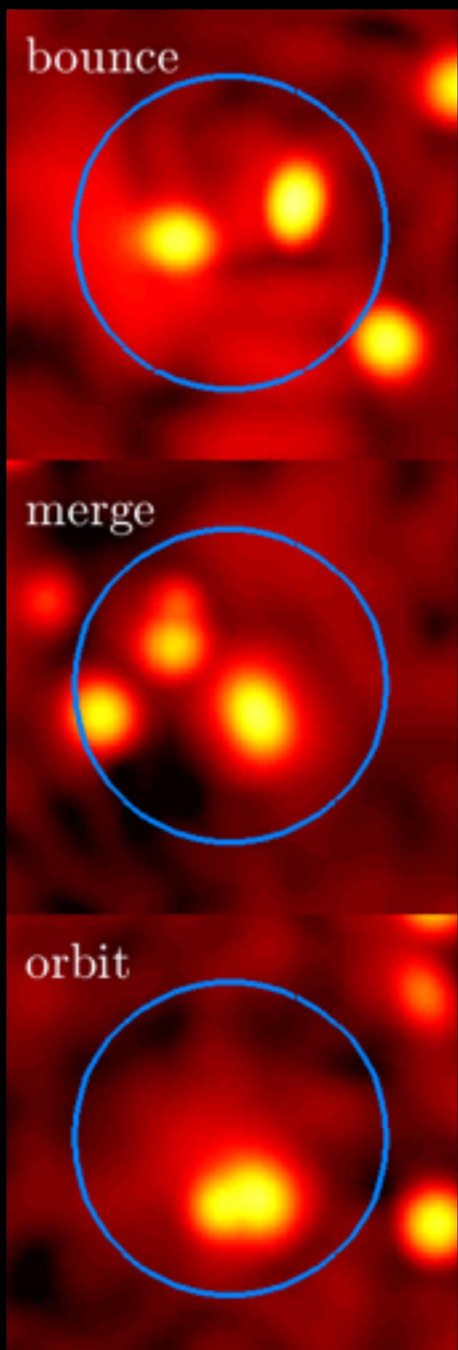

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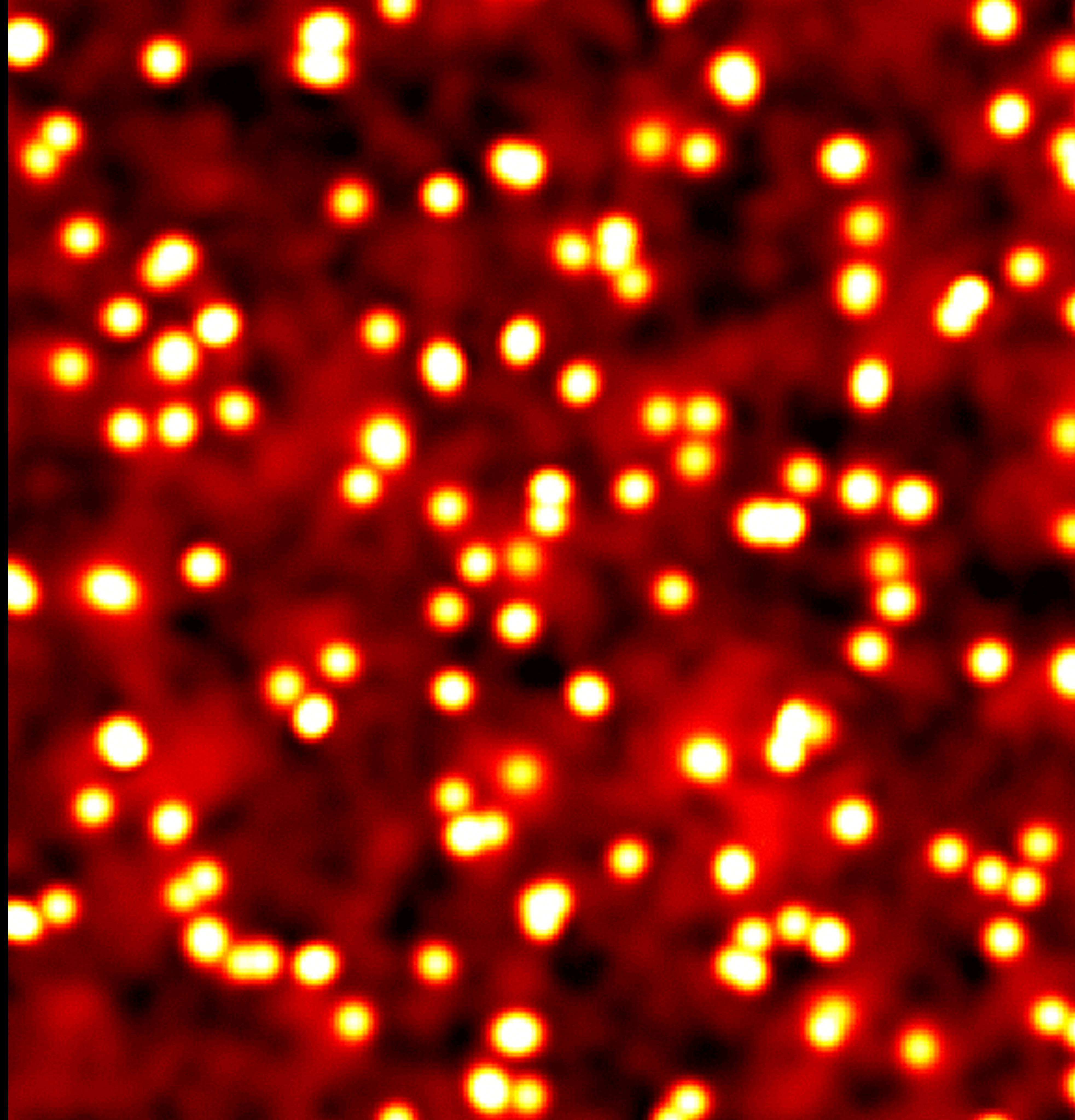
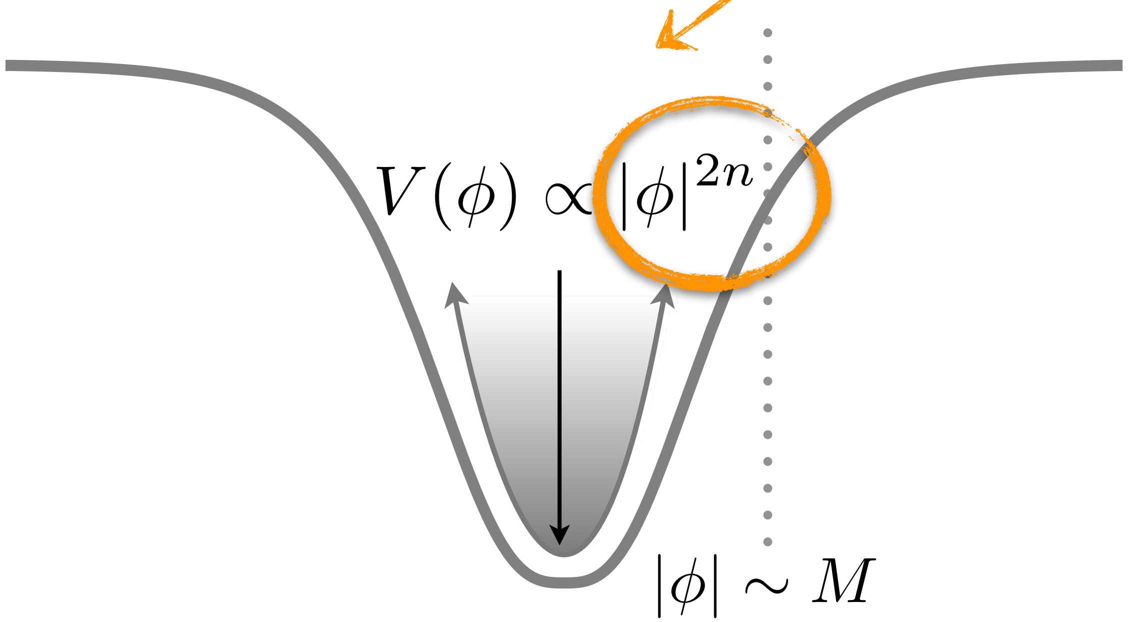


result 3: "equation-of-state"

the *spatially averaged* equation-of-state of fields

- $(n = 1)$ quadratic minima $w = 0$
- $(n > 1)$ non-quadratic minima $w = 1/3$ (after sufficient time)

power law at the minimum



$$w \equiv \frac{\langle p \rangle_s}{\langle \rho \rangle_s} = \frac{\langle \dot{\phi}^2/2 - (\nabla\phi)^2/6a^2 - V \rangle_s}{\langle \dot{\phi}^2/2 + (\nabla\phi)^2/2a^2 + V \rangle_s}$$

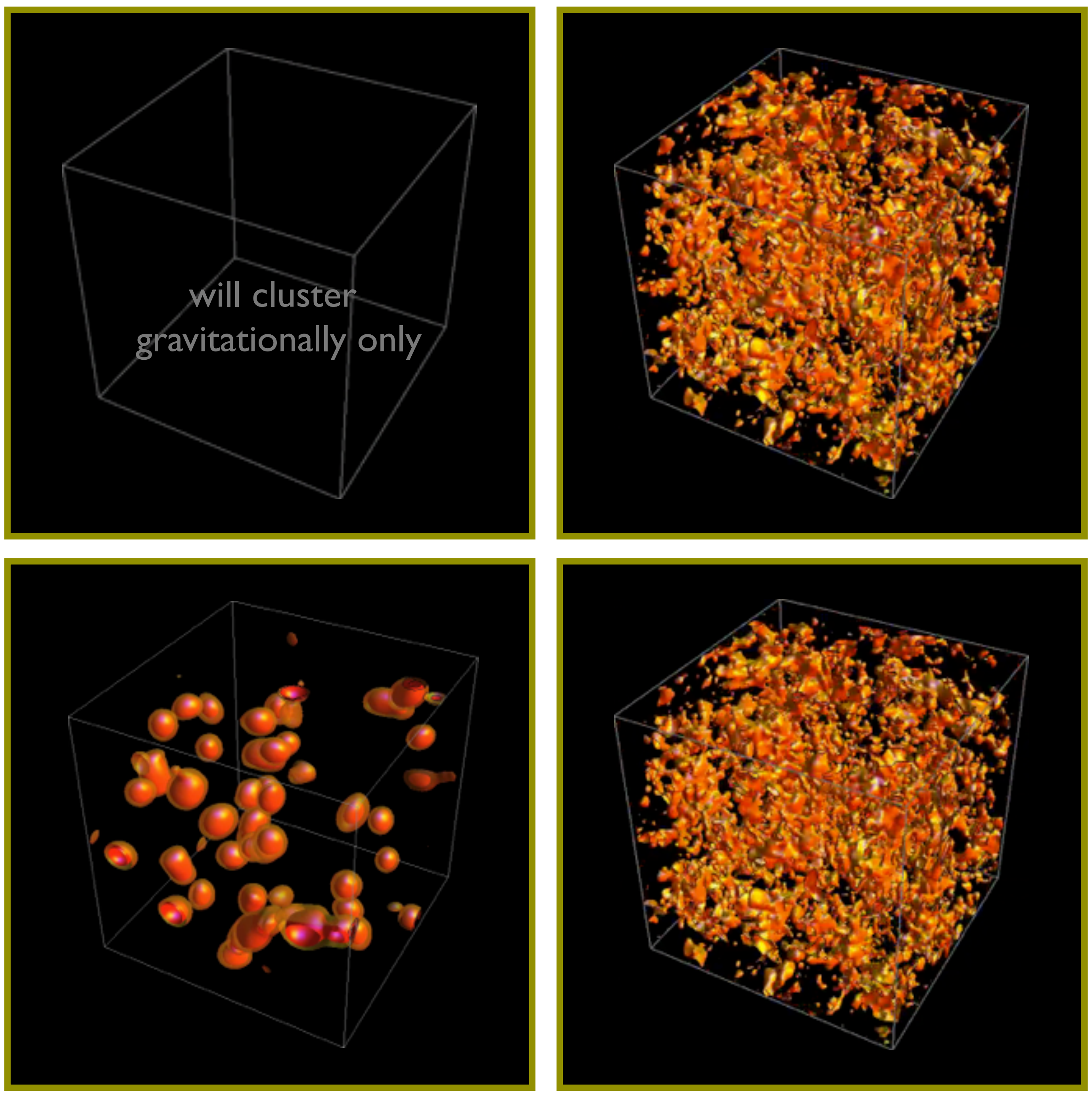
Lozanov & MA (2016/17)
1608.01213, 1710.06851

result 3: equation-of-state

$n = 1$ $\quad$ $n > 1$

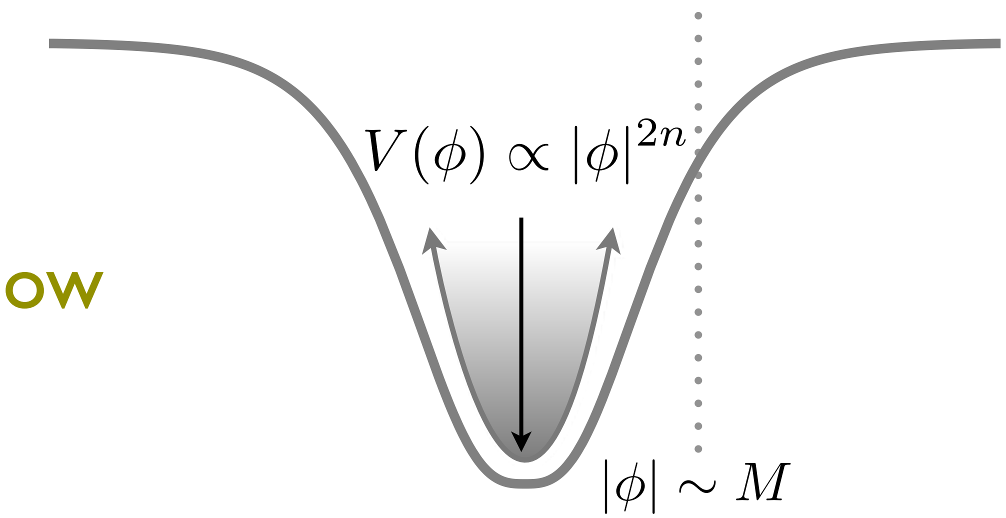
$M \sim m_{\rm pl}$

$M \ll m_{\rm pl}$



slow

fast



$$\boxed{w \to \begin{cases} 0 & \text{if } n = 1 \\ 1/3 & \text{if } n > 1 \end{cases}}$$

independent of M

$$w \neq \frac{n-1}{n+1}$$

Nonlinear Dynamics of Cosmological Scalar Fields

theoretical/numerical results

✓ 1. instability in oscillating fields

✓ 2. formation of solitons

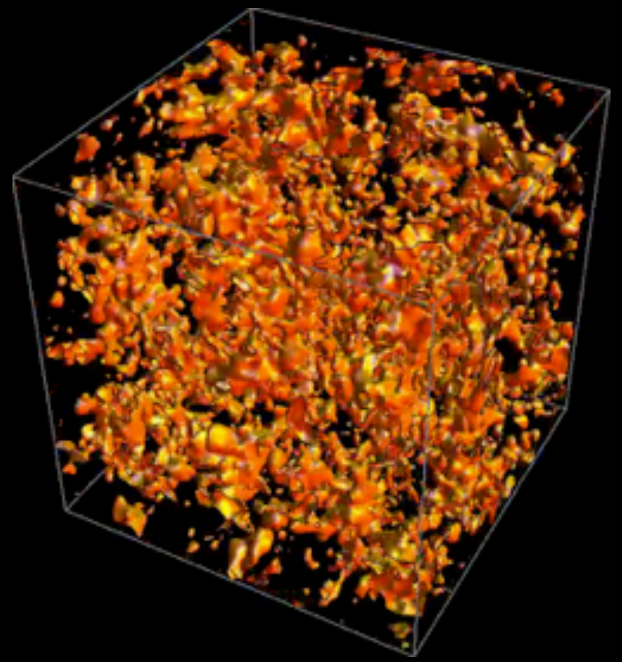
✓ 3. eq. of state

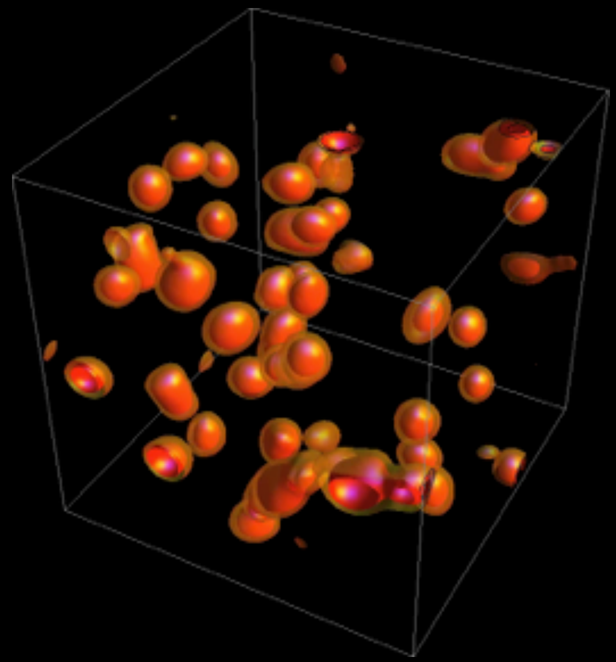
obs. implications

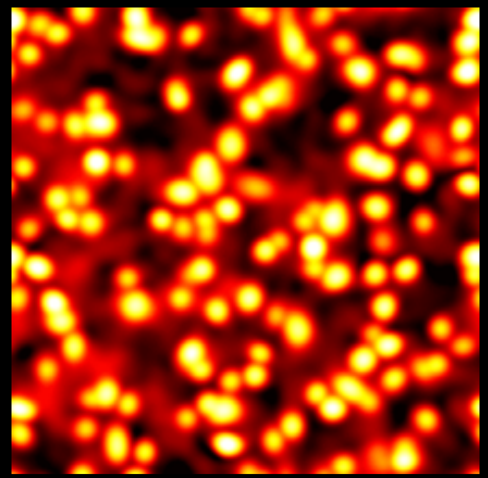
1. gravitational waves

2. structure formation

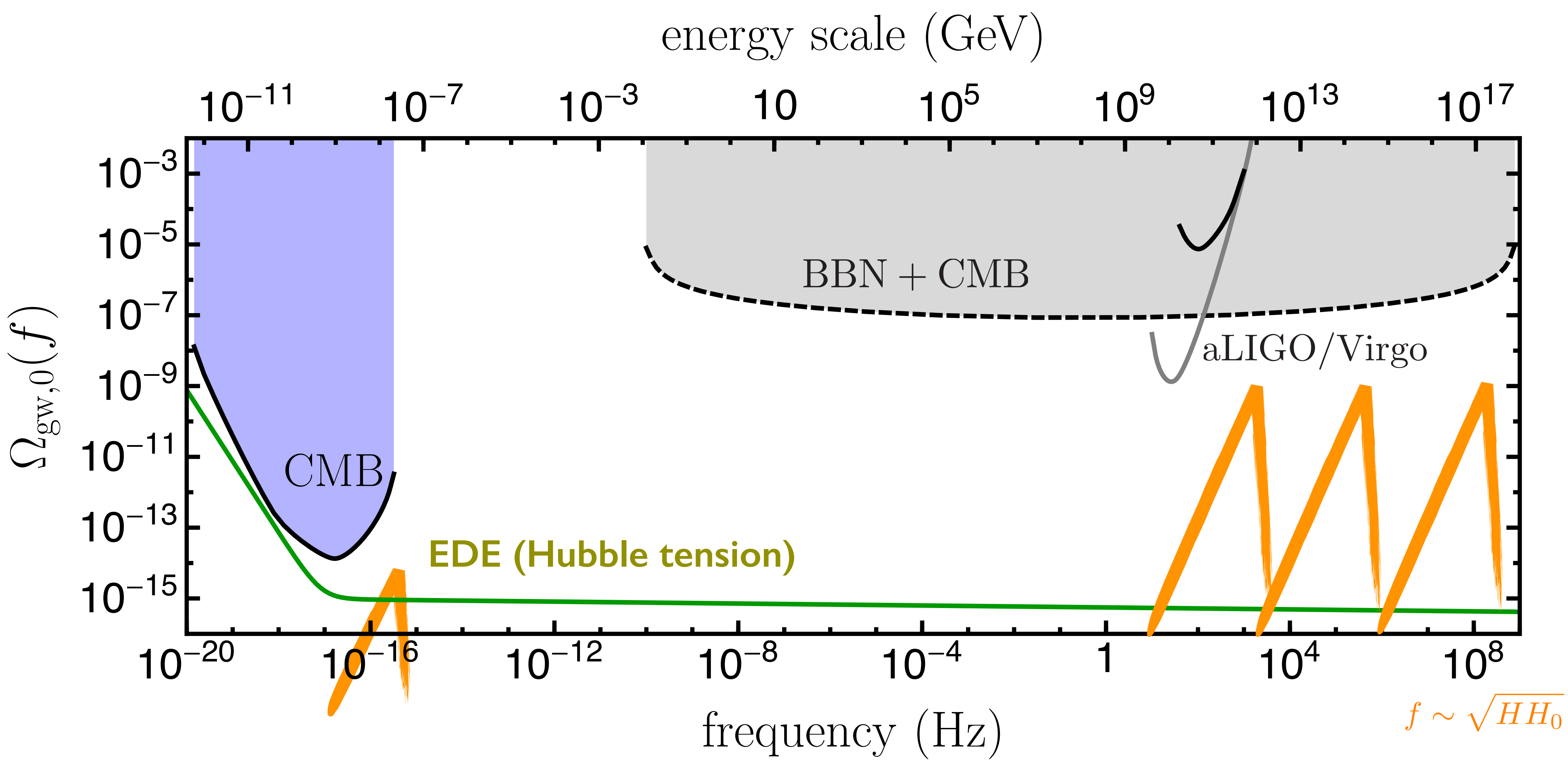
3. expansion history







implications: gravitational waves



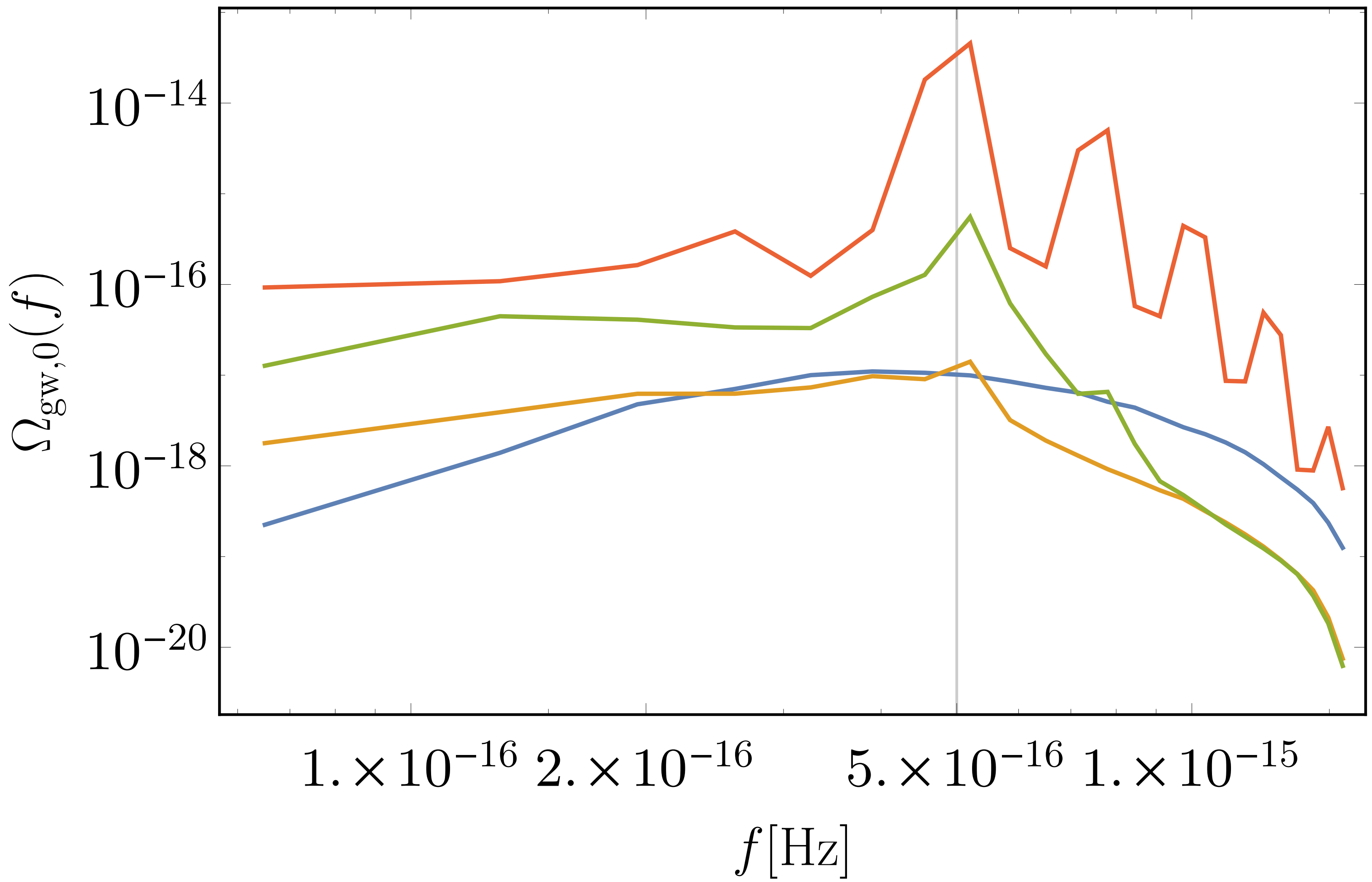
limits adapted from Lasky et. al (2015)

caveat* early universe g-waves amplitude depend on assumptions of expansion history

g-waves from nonlinear dynamics of early dark energy (*Hubble tension*)

ISW from g-waves in the CMB ?

ISW/lensing from scalar perturbations?



$$\frac{l(l+1)}{2\pi}C_l^{\mathrm{ISW}} \sim \mathcal{O}[1]\ [\mu\mathrm{K}]^2$$

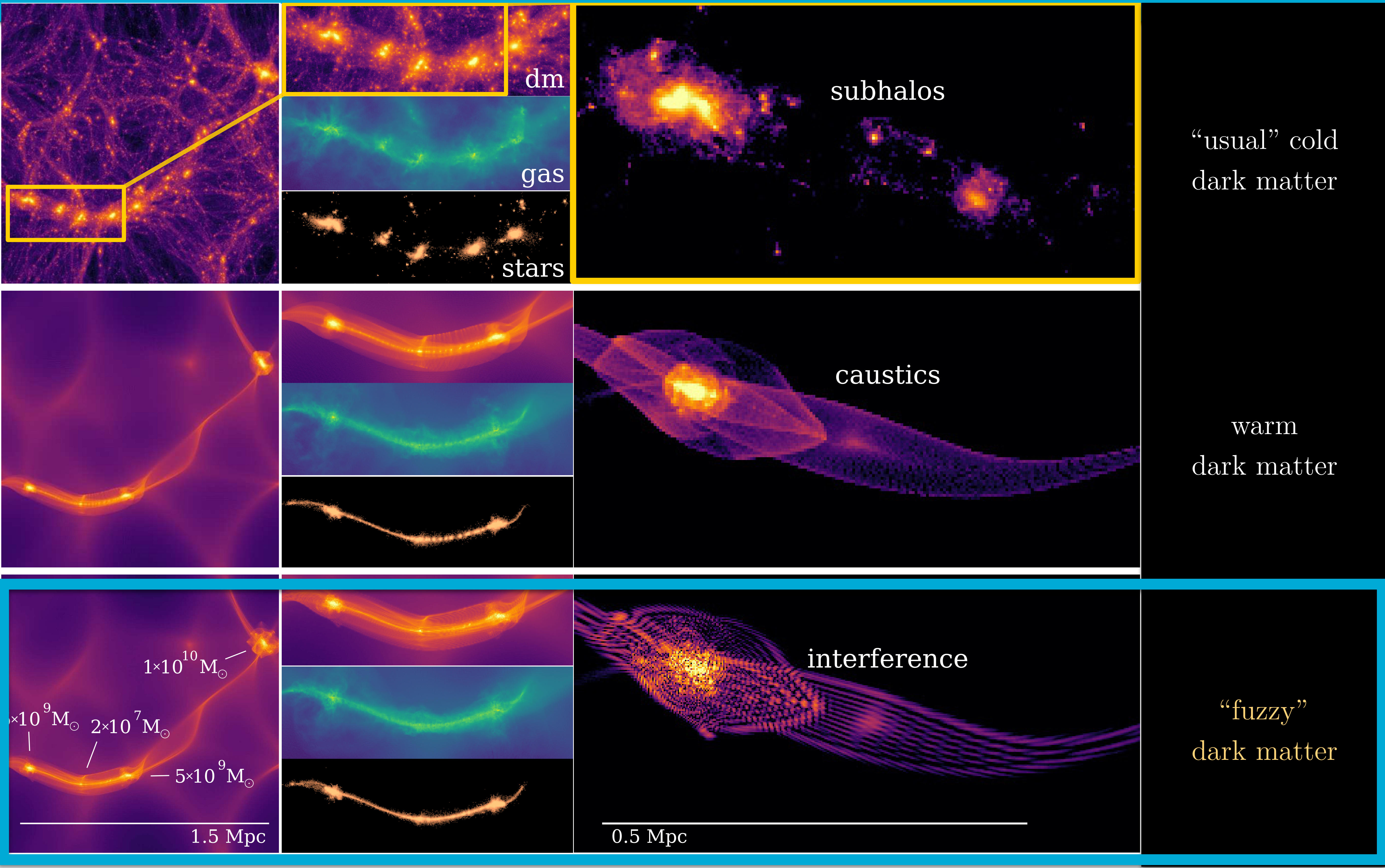
$$l \sim \mathrm{few} \times 10^3$$

MA, Lozanov & Smith (in progress)

Smith, Poulin & MA 1908.06995

structure formation with with baryons ("late" universe)

Mocz, Fialkov et. al (2019)

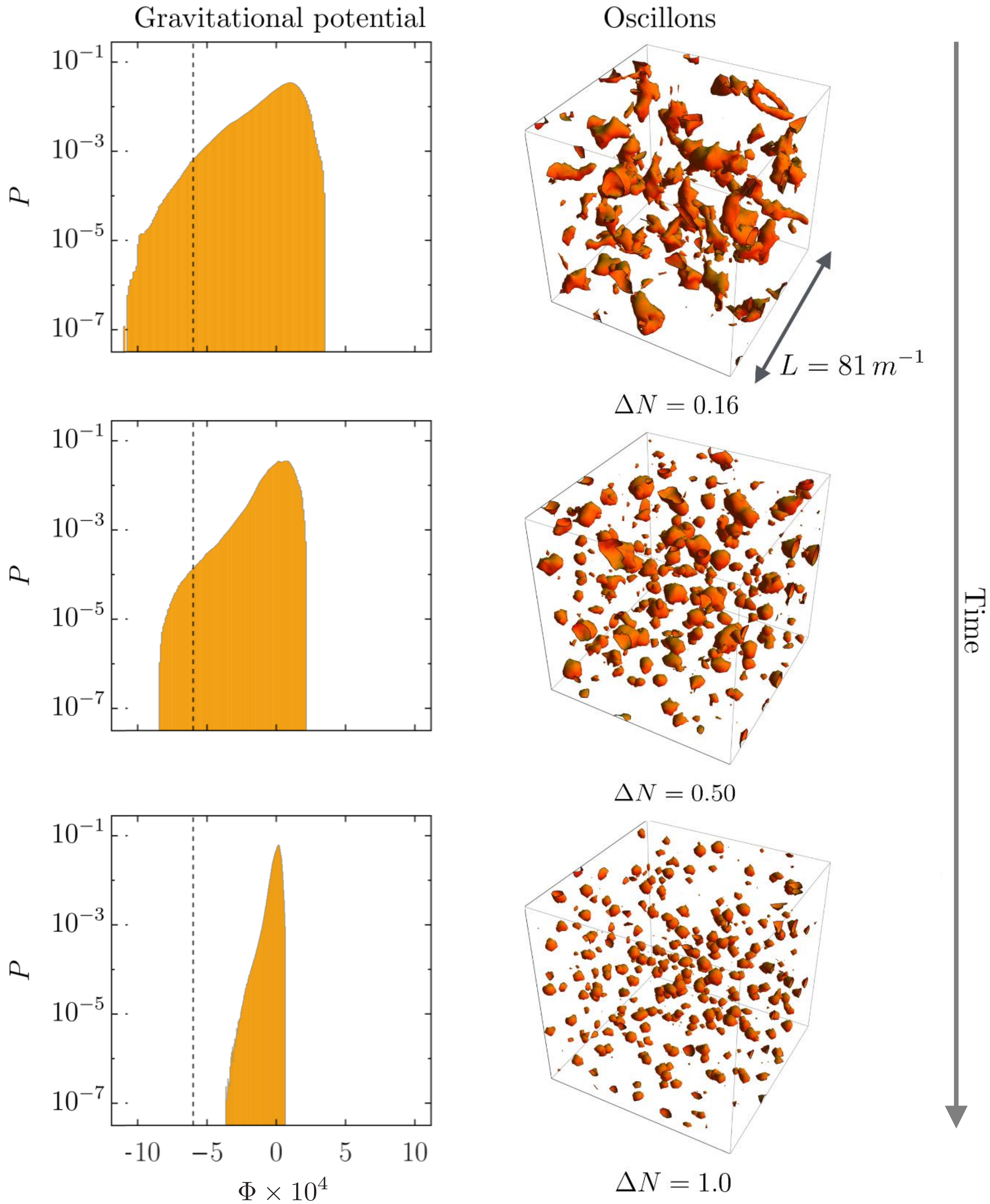


PBH formation from solitons?

$$\boxed{\Phi \lesssim \text{few} \times 10^{-3}}$$

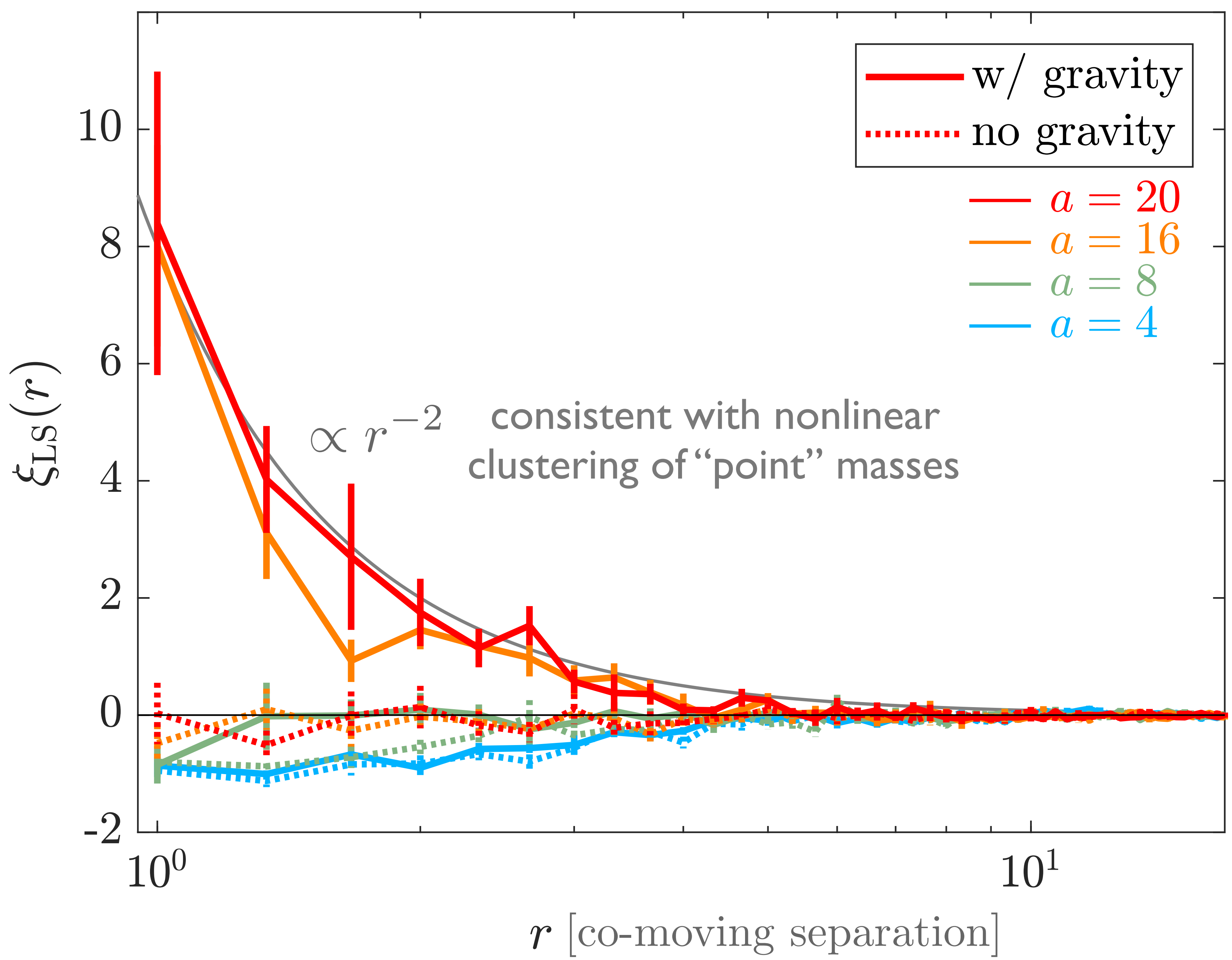
Not easy to form *PHBs*
from *individual solitons* from self resonance

However *accidental over-densities* in solitons *more likely* to form PBHs (Cotner et. al 2018/19)



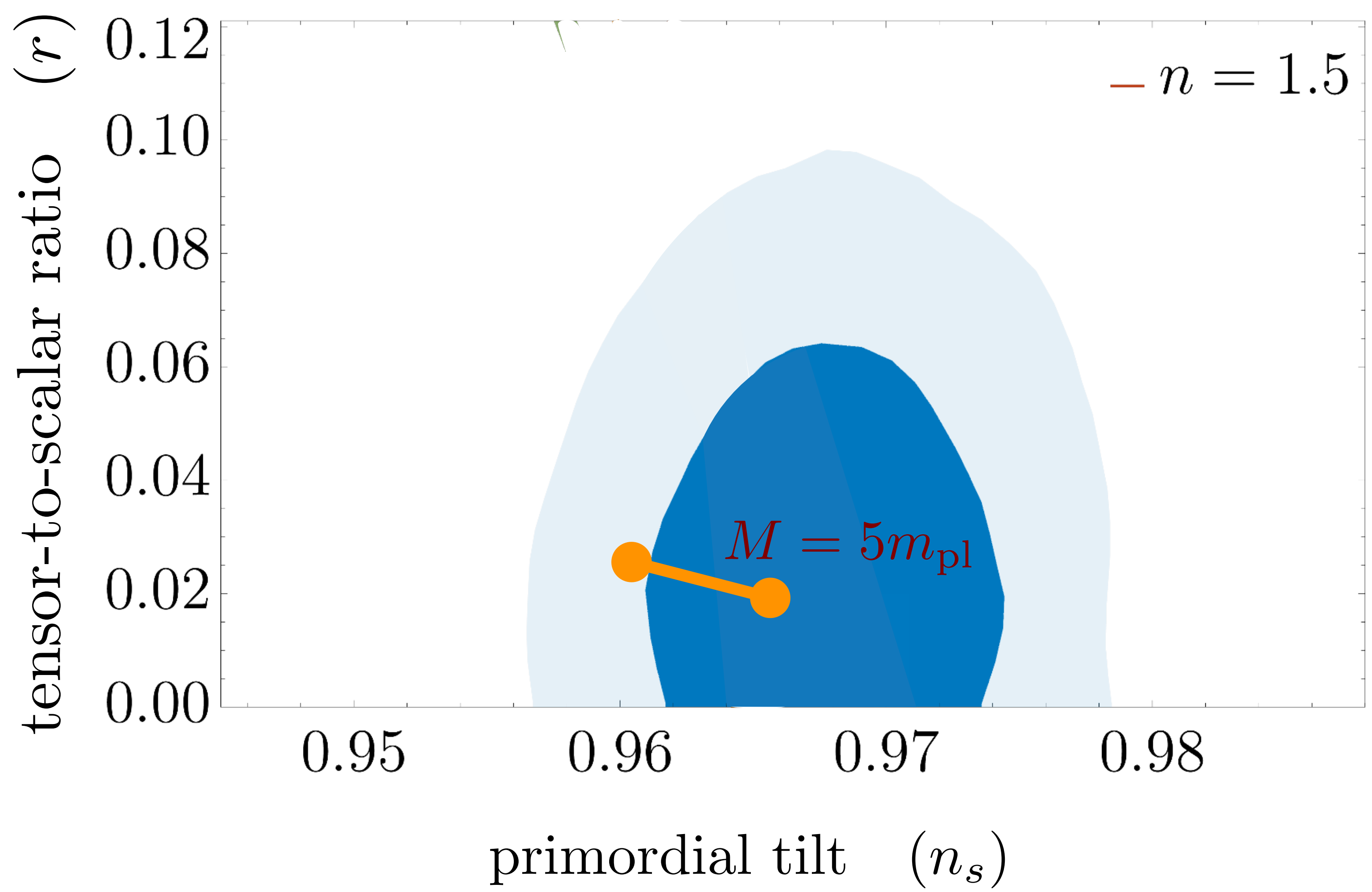
Lozanov & MA (2019)
1902.06736

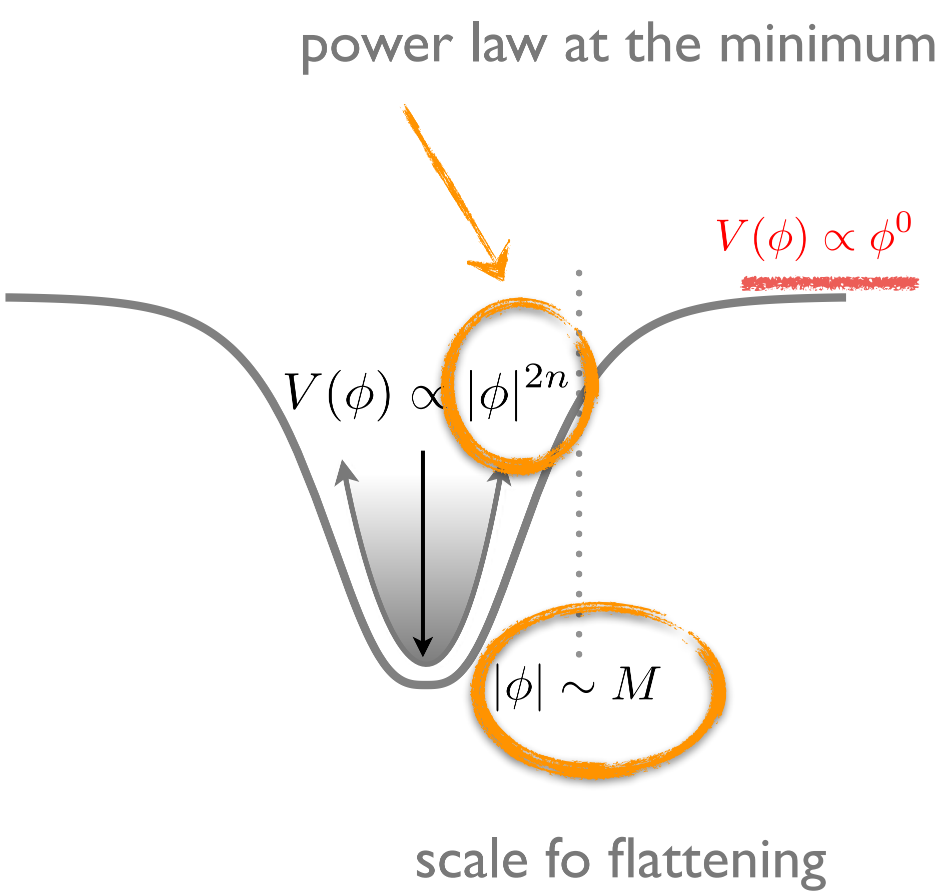
quantitative?
gravitational clustering of solitons



MA & Mocz (2019)

eq. of state & CMB observables



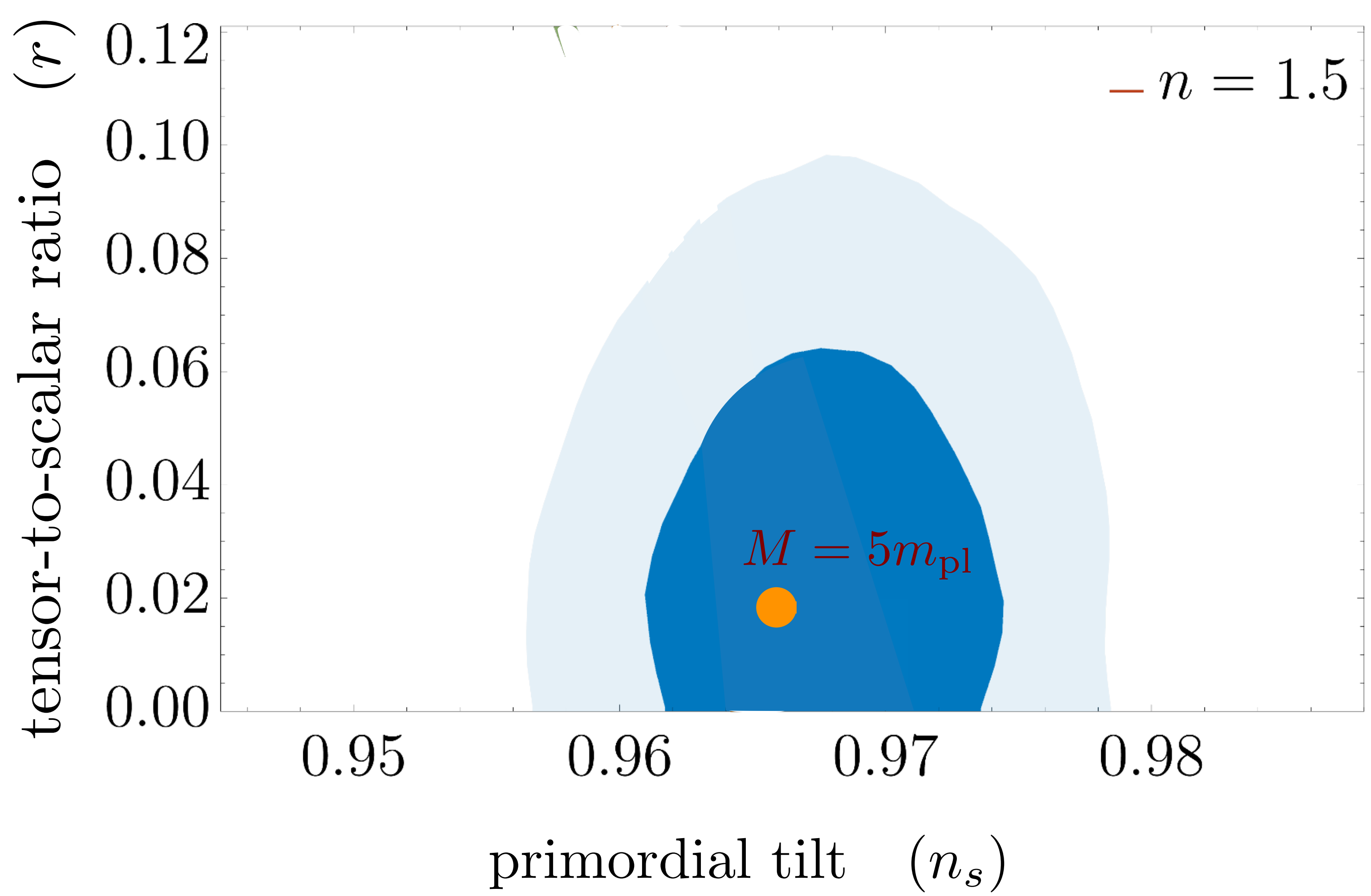


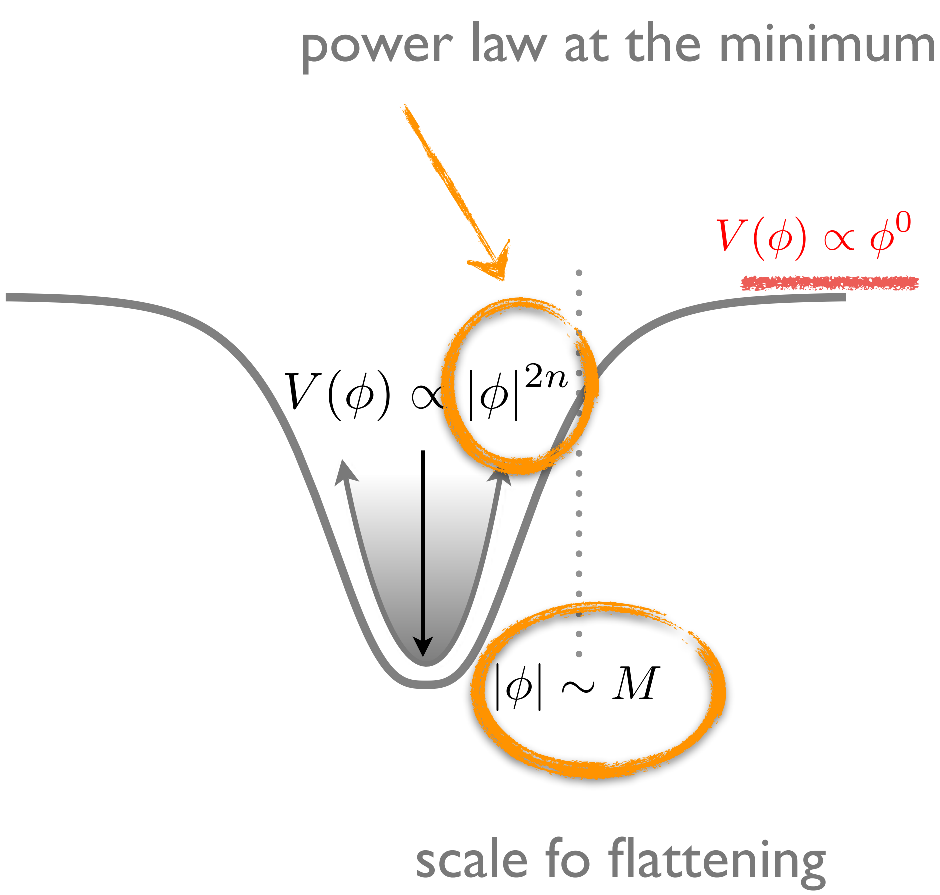
MA & Lozanov 2017 [1608.01213, 1710.06851]

also see: Kamionkowski & Munoz (2014)

* non-quadratic minimum

reduction in uncertainty





$$
\boxed{\Delta N_{\rm rad} \sim \begin{cases} 1 & M \lesssim 10^{-2} m_{\rm Pl}\,, \\ \dfrac{n+1}{3} \ln\left(\dfrac{\kappa}{\Delta\kappa}\dfrac{10M}{m_{\rm Pl}}\right) & M \gtrsim 10^{-2} m_{\rm Pl}\,. \end{cases}}
$$

Lozanov & MA (2017) [1608.01213, 1710.06851]
*all other fields are assumed to be light and massless.

* non-quadratic minimum

Nonlinear Dynamics of Cosmological Scalar Fields

3 theoretical/numerical results

- ✓ 1. instability in oscillating fields
- ✓ 2. formation of solitons
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3 obs. implications

- ✓ 1. gravitational waves
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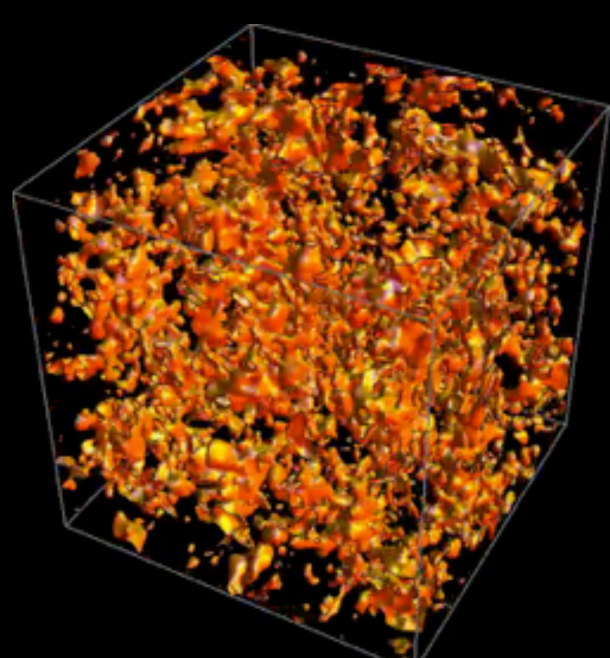
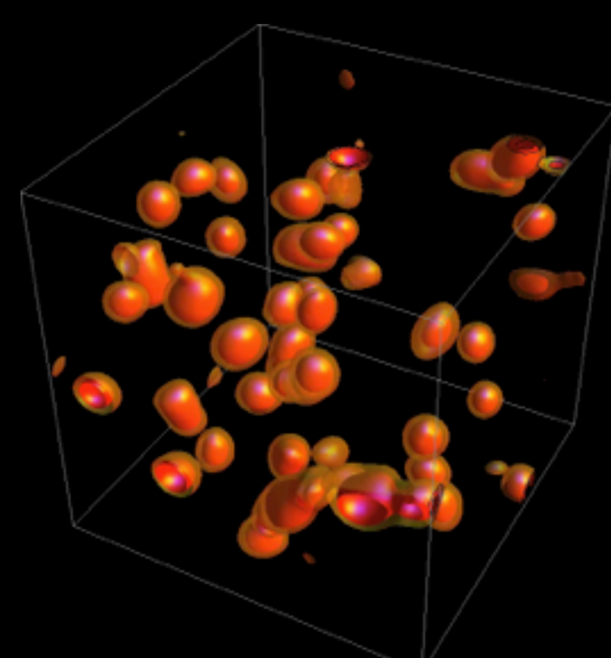
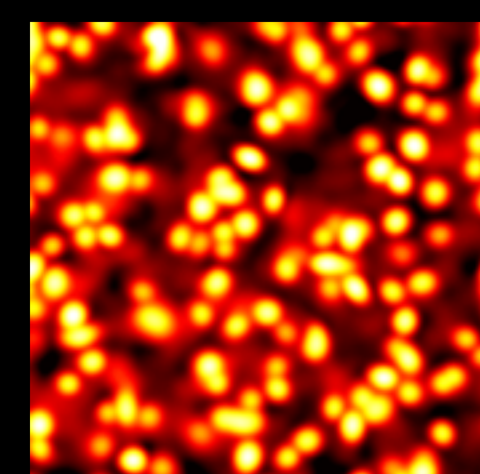






thanks

extra slides: one slide summaries

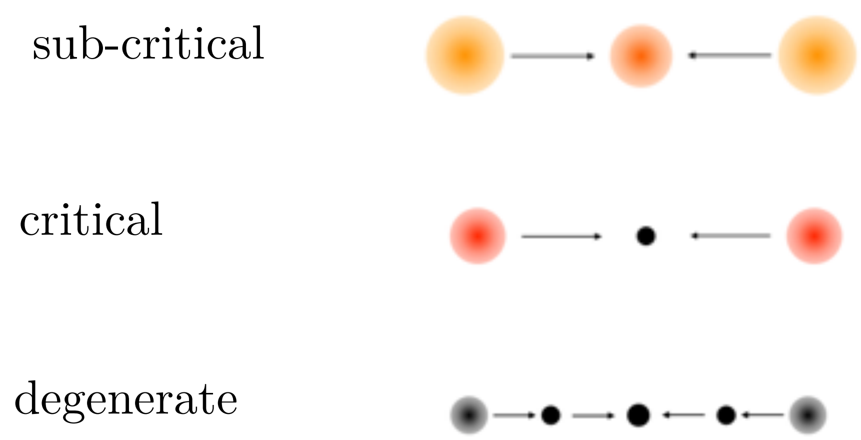
Numerical GR ✓

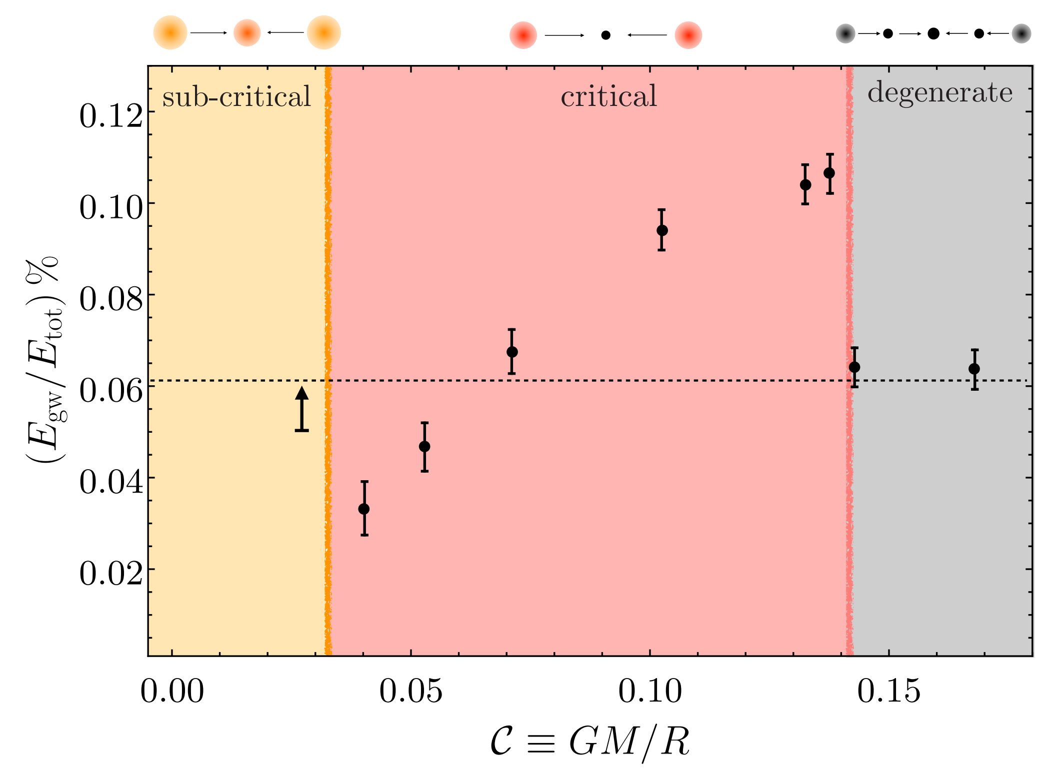
self-interactions ✗

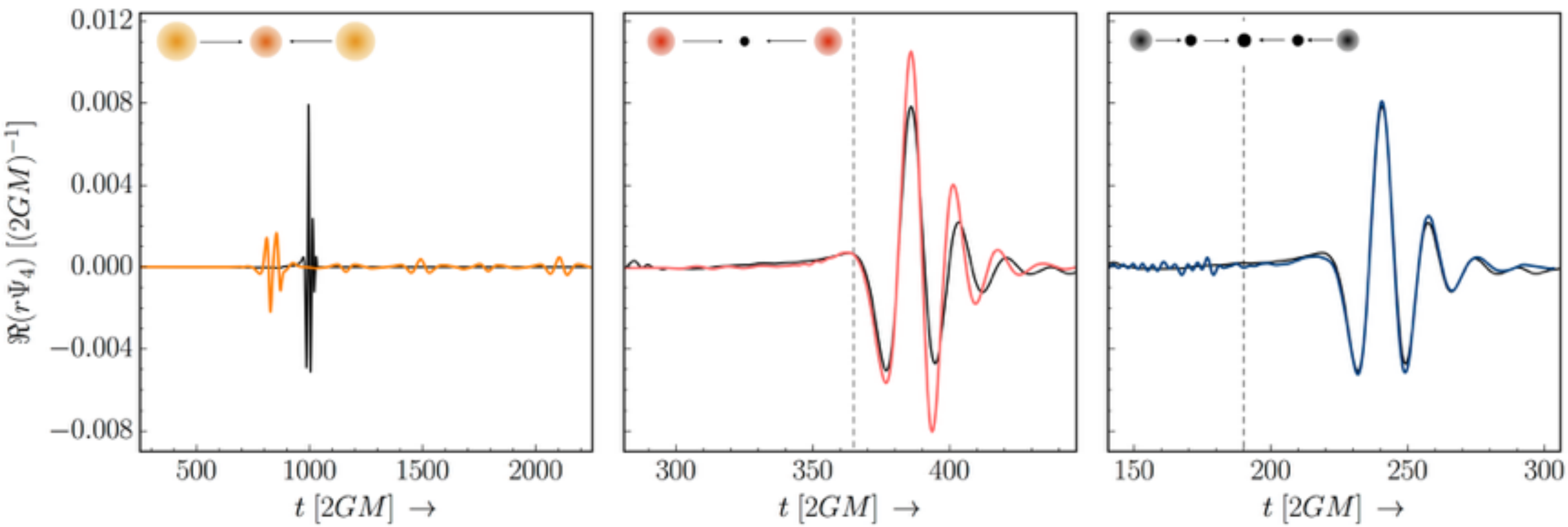
formation ✗

Helfer, Lim, Garcia & MA (2018)

gravitational waves from “ultra-compact” soliton collision

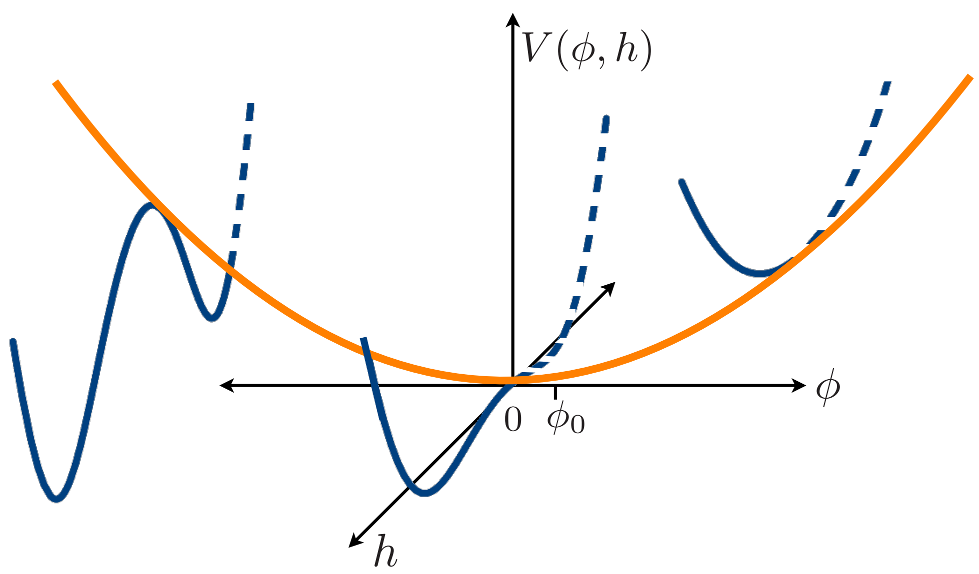






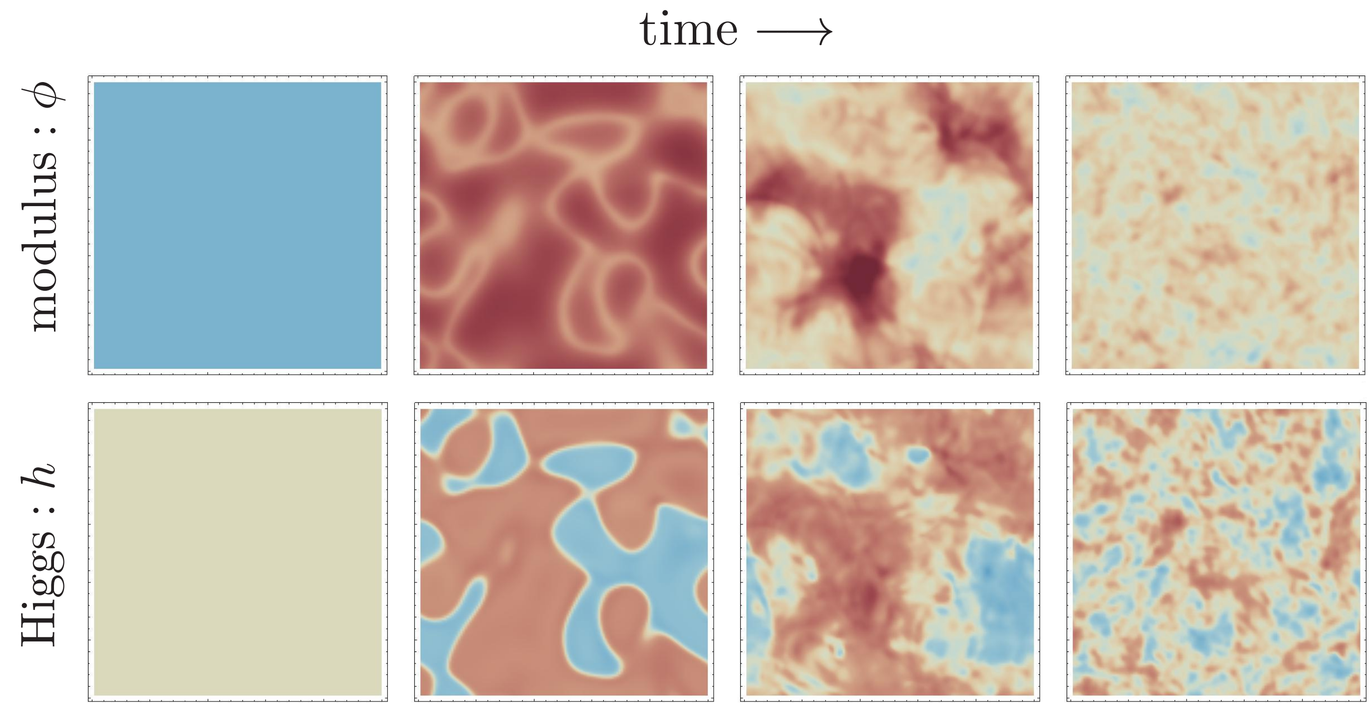
Nonlinear Dynamics of Higgs in the Early Universe

MA, J. Fan, K. Lozanov & M. Reece (2018)



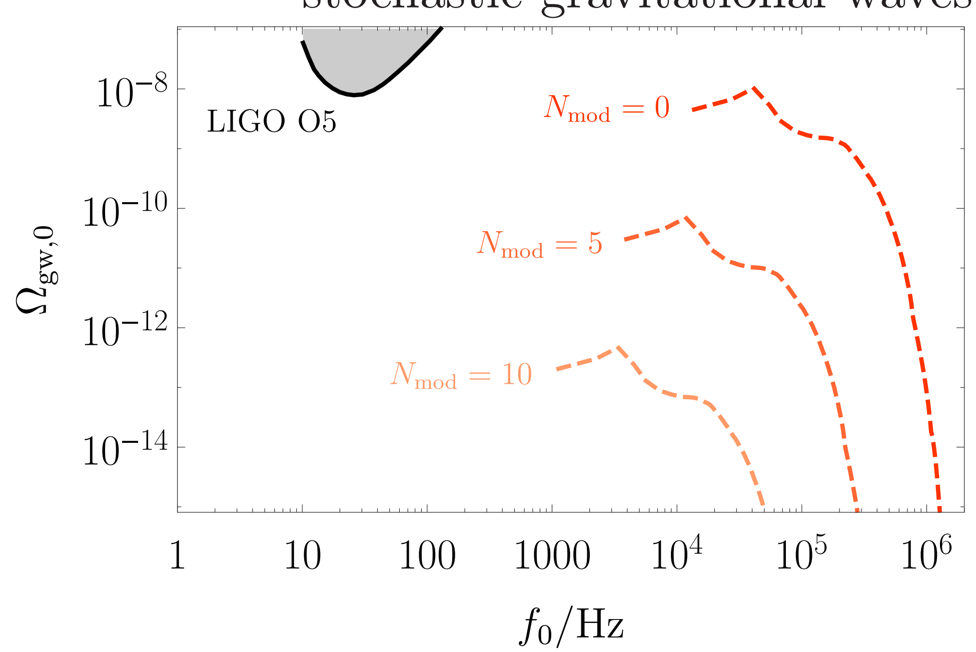
$$\frac{1}{2}m_\phi^2\phi^2 + \frac{M^2}{f}(\phi - \phi_0)\left(h^\dagger h - \frac{v^2}{2}\right) + \lambda(h^\dagger h)^2$$

fine tuning $\Leftrightarrow \dfrac{\phi_0}{f} \ll 1$



$$\frac{M^4}{2\lambda f^2 m_\phi^2} \to 1 \Leftrightarrow \text{rapid fragmentation}$$

stochastic gravitational waves



non-trivial eq. of state